

Unit-1

REAL NUMBERS
MCQs

Q.1. Choose the correct option.

1. $\sqrt{7}$ is:
(a) Integer (b) Rational number (c) Irrational number (d) Natural number
2. π and e are:
(a) Natural number (b) Integers (c) Rational number (d) Irrational number
3. If n is not a perfect square, then \sqrt{n} is:
(a) Rational number (b) Natural number (c) Integer (d) Irrational number
4. $\sqrt{3} + \sqrt{5}$ is:
(a) Whole number (b) Integer (c) Rational number (d) Irrational number
5. For all $x \in \mathbb{R}$, $x = x$ is called:
(a) Reflexive property (b) Transitive property
(c) Symmetric property (d) Trichotomy property
6. Let $a, b, c \in \mathbb{R}$ then $a > b$ or $b > c \Rightarrow a > c$ is called _____ property.
(a) Trichotomy (b) Transitive (c) Additive (d) Multiplicative
7. $2^x \times 8^x = 64$ then $x =$
(a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$
8. Let $a, b \in \mathbb{R}$ then $a = b$ and $b = a$ is called _____ property.
(a) Reflexive (b) Symmetric (c) Transitive (d) Additive
9. $\sqrt{75} + \sqrt{27} =$ _____
(a) $\sqrt{102}$ (b) $9\sqrt{3}$ (c) $5\sqrt{3}$ (d) $8\sqrt{3}$
10. The product of $(3 + \sqrt{5})(3 - \sqrt{5})$ is:
(a) Prime number (b) Odd number (c) Irrational number (d) Rational number
11. The value of $\frac{\sqrt{12} \times \sqrt{3}}{\sqrt{4}}$ is:
(a) 3 (b) 6 (c) 9 (d) 12
12. The solution to $2x + 5 = 15$ is:
(a) $x = 3$ (b) $x = 10$ (c) $x = 7.5$ (d) $x = 5$
13. Which of these is an irrational number?
(a) 0.333... (b) 3.14159... (c) 1.5 (d) 2332
14. Which number is rational?
(a) $\sqrt{3}$ (b) $\sqrt{16}$ (c) $\sqrt{5}$ (d) $\sqrt{7}$
15. Which property is illustrated by $a + b = b + a$?
(a) Associative (b) Identity (c) Distributive (d) Commutative
16. The simplified form of $\sqrt{50}$ is:
(a) $5\sqrt{2}$ (b) $10\sqrt{5}$ (c) $2\sqrt{5}$ (d) $5\sqrt{5}$
17. Which is a binomial surd?
(a) $\sqrt{3}$ (b) $\sqrt{3} + \sqrt{5}$ (c) $2\sqrt{3}$ (d) $\sqrt{9}$
18. Which is not a surd?
(a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{16}$ (d) $\sqrt{18}$
19. Which of the following is an example of a rational number?
(a) π (b) $\frac{3}{4}$ (c) $\sqrt{2}$ (d) e
20. What is an example of a recurring decimal?
(a) 3.14159265... (b) 0.3333... (c) 2.716281828... (d) $\sqrt{7}$
21. The reciprocal of $\sqrt{7}$ is:
(a) $\frac{1}{\sqrt{7}}$ (b) $\sqrt{7}$ (c) 7 (d) $7\sqrt{7}$

22. If $\sqrt{x} = 5$, then $x = ?$
 (a) 10 (b) 25 (c) 5 (d) 15
23. The solution to $2^x = 16$ is:
 (a) 6 (b) 8 (c) 2
24. Which is equivalent to $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$?
 (a) $5 + 2\sqrt{6}$ (b) 1 (c) $\sqrt{6}$ (d) $5 - 2\sqrt{6}$
25. Which of the following is not an integer?
 (a) -3 (b) 0 (c) 4.5 (d) 7

Unit-1

REAL NUMBERS
Short Questions

Exercise 1.1

1. State symmetric property of real numbers.

Ans: Symmetric property:

$$\forall a, b \in R, a = b \Rightarrow b = a$$

2. Define set of Real number.

Ans: Real number: The set of Real numbers is the union of the set of rational numbers and irrational numbers i.e.,
 $R = Q \cup Q'$

3. Define Terminating decimal numbers.

Ans: Terminating decimal numbers: A decimal number with a finite number of digits after the decimal point is called a terminating decimal number.

4. Give any two examples of irrational numbers?

Ans: Two examples of irrational numbers are $\sqrt{2}, \pi$.

5. Is 0 a rational number? Explain.

Ans: Yes, 0 is a rational number because it can be written in the form of $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$.

Example: $0 = \frac{0}{7}$ where $0, 7 \in Z$ and $7 \neq 0$

6. State trichotomy property of real numbers.

Ans: Trichotomy property:

$$\forall a, b \in R \Rightarrow a = b \text{ or } a > b \text{ or } a < b$$

7. Find two rational numbers between 4 and 5.

Solution:

For this find the average of 4 and 5

$$\frac{4+5}{2} = \frac{9}{2}$$

So, $\frac{9}{2}$ is a rational number between 4 and 5

Now, find the average of $\frac{9}{2}$ and 5.

$$= \left(\frac{9}{2} + 5\right) \div 2$$

$$= \left(\frac{9+10}{2}\right) \div 2$$

$$= \frac{19}{2} \times \frac{1}{2} = \frac{19}{4}$$

Hence two rational numbers between 4 and 5 are $\frac{9}{2}$ and $\frac{19}{4}$.

8. Express the following recurring decimals as the rational number $\frac{p}{q}$, where p and q are integers. $0.\overline{93}$

Solution:

Let $x = 0.\overline{93}$,

$$x = 0.939393 \dots$$

Multiply by 100 on both sides

$$100x = 100(0.939393 \dots)$$

$$100x = 93.939393 \dots$$

$$100x = 93 + 0.939393 \dots$$

$$100x = 93 + 0.93$$

$$100x = 93 + x$$

$$100x - x = 93$$

$$99x = 93$$

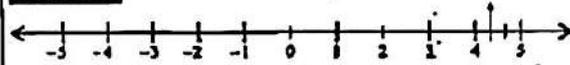
$$x = \frac{93}{99}$$

$$\Rightarrow 0.\overline{93} = \frac{93}{99}$$

Which shows rational number in the form of $\frac{p}{q}$.

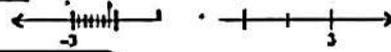
9. Represent the on number line: $4\frac{1}{3}$

Solution:



10. Represent the on number line: $-2\frac{1}{7}$

Solution:



Exercise 1.2

11. Write any two laws of Indices.

Ans: Two laws of Indices:

(i) $a^m \cdot a^n = a^{m+n}$

(ii) $(a^m)^n = a^{m \cdot n}$

12. Define surd.

Ans: Surd: An irrational radical with rational radicand is called a surd.

13. Define binomial surd.

Ans: Binomial surd: A surd that contains the sum of two monomial surds is called a binomial surd e.g., $\sqrt{3} + \sqrt{5}, \sqrt{2} + \sqrt{7}$.

4. Rationalize the denominator: $\frac{13}{4 + \sqrt{3}}$

Solution:

$$\begin{aligned} & \frac{13}{4 + \sqrt{3}} \\ &= \frac{13}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}} \\ &= \frac{13(4 - \sqrt{3})}{(4)^2 - (\sqrt{3})^2} \\ &= \frac{13(4 - \sqrt{3})}{16 - 3} \\ &= \frac{13(4 - \sqrt{3})}{13} \\ &= 4 - \sqrt{3} \end{aligned}$$

15. Rationalize the denominator: $\frac{\sqrt{2} + \sqrt{5}}{\sqrt{3}}$

Solution:

$$\begin{aligned} & \frac{\sqrt{2} + \sqrt{5}}{\sqrt{3}} \\ &= \frac{\sqrt{2} + \sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{3}(\sqrt{2} + \sqrt{5})}{(\sqrt{3})^2} \end{aligned}$$

$$\begin{aligned} &= \frac{\sqrt{3}(\sqrt{2} + \sqrt{5})}{3} = \frac{\sqrt{2 \times 3} + \sqrt{5 \times 3}}{3} \\ &= \frac{\sqrt{6} + \sqrt{15}}{3} \\ &= \frac{1}{3}(\sqrt{6} + \sqrt{15}) \end{aligned}$$

16. Rationalize the denominator: $\frac{\sqrt{2} - 1}{\sqrt{5}}$

Solution:

$$\begin{aligned} & \frac{\sqrt{2} - 1}{\sqrt{5}} \\ &= \frac{\sqrt{2} - 1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{5}(\sqrt{2} - 1)}{(\sqrt{5})^2} \\ &= \frac{\sqrt{5}(\sqrt{2} - 1)}{5} \\ &= \frac{\sqrt{2 \times 5} - \sqrt{5}}{5} \\ &= \frac{\sqrt{10} - \sqrt{5}}{5} \\ &= \frac{1}{5}(\sqrt{10} - \sqrt{5}) \end{aligned}$$

17. Simplify: $\sqrt[3]{27x^6y^9z^3}$

Solution:

$$\begin{aligned} & \sqrt[3]{27x^6y^9z^3} \\ &= (27x^6y^9z^3)^{\frac{1}{3}} \quad \because \sqrt[n]{a} = a^{\frac{1}{n}} \\ &= (3^3x^6y^9z^3)^{\frac{1}{3}} \\ &= (3^3)^{\frac{1}{3}}(x^6)^{\frac{1}{3}}(y^9)^{\frac{1}{3}}(z^3)^{\frac{1}{3}} \quad \because (ab)^n = a^n b^n \\ &= 3^{3 \times \frac{1}{3}} \cdot x^{6 \times \frac{1}{3}} \cdot y^{9 \times \frac{1}{3}} \cdot z^{3 \times \frac{1}{3}} \\ &= 3x^2y^3z \end{aligned}$$

18. Simplify: $(64)^{-\frac{4}{3}}$

Solution:

$$\begin{aligned} & (64)^{-\frac{4}{3}} \\ &= \frac{1}{(64)^{\frac{4}{3}}} = \frac{1}{(4^3)^{\frac{4}{3}}} = \frac{1}{4^{\frac{3 \cdot 4}{3}}} \\ &= \frac{1}{4^4} = \frac{1}{256} \end{aligned}$$

19. Simplify: $\sqrt[3]{64x^6}$

Solution:

$$\begin{aligned} & \sqrt[3]{64x^6} \\ &= \sqrt[3]{64(1)} \\ &= \sqrt[3]{64} \\ &= \sqrt[3]{2^4 \cdot 2^2} = \sqrt[3]{2^6} \cdot \sqrt[3]{2^2} \\ &= 2\sqrt[3]{4} \end{aligned}$$

20. Simplify: $\sqrt[3]{\frac{x^{15}y^{33}}{z^{20}}}$

Solution:

$$\begin{aligned} & \sqrt[3]{\frac{x^{15}y^{33}}{z^{20}}} \\ &= \left(\frac{x^{15}y^{33}}{z^{20}}\right)^{\frac{1}{3}} \\ &= \frac{x^{15 \cdot \frac{1}{3}} y^{33 \cdot \frac{1}{3}}}{z^{\frac{20 \cdot 1}{3}}} \\ &= \frac{x^5 y^9}{z^{\frac{20}{3}}} \end{aligned}$$

21. Simplify: $\frac{6(3)^{n+2}}{3^{n+1} - 3^n}$

Solution:

$$\begin{aligned} & \frac{6(3)^{n+2}}{3^{n+1} - 3^n} \\ &= \frac{6 \cdot 3^n \cdot 3^2}{3^n \cdot 3^1 - 3^n} \\ &= \frac{6 \times 3^n \times 9}{3^n(3-1)} = \frac{54 \times 3^n}{3^n(2)} \\ &= 27 \end{aligned}$$

22. Simplify: $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

Solution:

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}}$$

$$\begin{aligned} &= \left(\frac{16}{81}\right)^{\frac{3}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}} \\ &= \frac{2^{4 \cdot \frac{3}{4}}}{3^{4 \cdot \frac{3}{4}}} \\ &= \frac{2^3}{3^3} \\ &= \frac{8}{27} \end{aligned}$$

23. Simplify: $\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

Solution:

$$\begin{aligned} & \frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}} \\ &= \frac{3^n \times 9^n \times 9^1}{3^n \times 3^{-1} \times 9^n \times 9^{-1}} \\ &= \frac{9}{3^{-1} \times 9^{-1}} \\ &= 9 \times 3 \times 9 \\ &= 243 \end{aligned}$$

24. Simplify: $\frac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 2^2 \times 5^n}$

Solution:

$$\begin{aligned} & \frac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 2^2 \times 5^n} \\ &= \frac{5^n \times 5^3 - 6.5^n \times 5}{9 \times 5^n - 2^2 \times 5^n} \\ &= \frac{5^n(5^3 - 30)}{5^n(9 - 2^2)} \\ &= \frac{125 - 30}{9 - 4} \\ &= \frac{95}{5} = 19 \end{aligned}$$

25. If $x = 3 + \sqrt{8}$ then find the value of

$$x - \frac{1}{x}$$

Solution:

$$\begin{aligned} & x = 3 + \sqrt{8} \\ & \frac{1}{x} = \frac{1}{3 + \sqrt{8}} \\ &= \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}} \\ &= \frac{3 - \sqrt{8}}{(3)^2 - (\sqrt{8})^2} \end{aligned}$$

$$= \frac{3-\sqrt{8}}{9-8}$$

$$\frac{1}{x} = 3-\sqrt{8}$$

$$\begin{aligned} x - \frac{1}{x} &= (3+\sqrt{8}) - (3-\sqrt{8}) \\ &= 3+\sqrt{8}-3+\sqrt{8} \\ &= 2\sqrt{8} \end{aligned}$$

26. The sum of two real numbers is 8, and their difference is 2. Find the numbers.

Solution:

Let a and b be two real numbers then

$$a + b = 8 \quad \dots(i)$$

$$a - b = 2 \quad \dots(ii)$$

Add eq. (i) and eq. (ii)

$$a + b = 8$$

$$a - b = 2$$

$$2a = 10$$

$$a = \frac{10}{2} = 5$$

Putting the value of a in equation (i)

$$a + b = 8$$

$$5 + b = 8$$

$$b = 8 - 5$$

$$b = 3$$

Thus, required real numbers 5, 3.

27. Rationalize the denominator: $\frac{3}{\sqrt{5}-\sqrt{3}}$

Solution:

$$\begin{aligned} \frac{3}{\sqrt{5}-\sqrt{3}} &= \frac{3}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\ &= \frac{3(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} = \frac{3(\sqrt{5}+\sqrt{3})}{5-3} \\ &= \frac{3(\sqrt{5}+\sqrt{3})}{2} \end{aligned}$$

28. Simplify: $(0.027)^{-\frac{1}{3}}$

Solution:

$$(0.027)^{-\frac{1}{3}}$$

$$= \left(\frac{27}{1000}\right)^{-\frac{1}{3}}$$

$$= \left(\frac{1000}{27}\right)^{\frac{1}{3}}$$

$$= \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}}$$

$$= \frac{10^{3 \cdot \frac{1}{3}}}{3^{3 \cdot \frac{1}{3}}}$$

$$= \frac{10}{3}$$

29. Simplify: $\sqrt[3]{16x^4y^8}$

Solution:

$$\sqrt[3]{16x^4y^8}$$

$$= (16x^4y^8)^{\frac{1}{3}}$$

$$= (2^4x^4y^8)^{\frac{1}{3}}$$

$$= 2^{4 \cdot \frac{1}{3}} \times x^{4 \cdot \frac{1}{3}} \times y^{8 \cdot \frac{1}{3}}$$

$$= 2xy^2$$

$$\sqrt[3]{a} = a^{\frac{1}{3}}$$

$$\therefore (a^m)^n = a^{mn}$$

Unit-1

REAL NUMBERS

Long Questions

- Simplify: $\sqrt{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^7}}$
- Simplify: $\frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}}$
- Simplify: $\frac{(16)^{n+1} + 20(4^{2n})}{2^{n-3} \times 8^{n+1}}$
- If $x = 3 + \sqrt{8}$ then find the value of $x^4 + \frac{1}{x^4}$

5. If $x = 3 + \sqrt{8}$ then find the value of $\left(x - \frac{1}{x}\right)^4$
6. If $x = 3 + \sqrt{8}$ then find the value of $x^3 - \frac{1}{x^3}$
7. Find the rational numbers p and q such that $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$
8. Simplify: $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{2}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$
9. Simplify: $\frac{54 \times \sqrt[3]{(27)^{2a}}}{9^{a+1} + 216(3^{2a-1})}$
10. Simplify: $\left(\frac{3}{4}\right)^{-2} - \left(\frac{4}{9}\right)^3 \times \frac{16}{27}$
11. Simplify: $\sqrt{\frac{(216)^{\frac{1}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}}}}$

Unit-2

**LOGARITHMS
MCQs**

Q.1: Choose the correct option.

- The standard form of 5.2×10^4 is:
(a) 52,000 (b) 520,000 (c) 5,200,000 (d) 52,000,000
- Scientific notation of 0.00034 is:
(a) 3.4×10^3 (b) 3.4×10^{-4} (c) 3.4×10^4 (d) 3.4×10^{-3}
- The base of common logarithm is:
(a) 2 (b) 10 (c) 5 (d) e
- The base of natural logarithm is:
(a) e (b) 10 (c) 2 (d) π
- $\log 100 =$ _____
(a) 2 (b) 3 (c) 10 (d) 1
- If $\log 2 = 0.3010$, then $\log 200$ is:
(a) 1.3010 (b) 0.6010 (c) 2.3010 (d) 2.6010
- $\log (0) =$ _____
(a) positive (b) negative (c) zero (d) undefined
- $\log 10,000 =$ _____
(a) 2 (b) 3 (c) 4 (d) 5
- $\log 5 + \log 3 =$ _____
(a) $\log 0$ (b) $\log 2$ (c) $\log \left(\frac{5}{3}\right)$ (d) $\log 15$
- $3^4 = 81$ in logarithmic form is:
(a) $\log_3 4 = 81$ (b) $\log_4 3 = 81$
(c) $\log_3 81 = 4$ (d) $\log_4 81 = 3$
- Convert 45,000 to scientific notation.
(a) 4.5×10^3 (b) 4.5×10^4 (c) 45×10^3 (d) 0.45×10^5
- Convert 9.01×10^4 to ordinary notation.
(a) 9010 (b) 90100 (c) 901000 (d) 901

13. 73×10^3 in scientific notation is:
 (a) 7.3×10^2 (b) 7.3×10^3 (c) 7.3×10^4 (d) 73×10^3
14. Convert 6.7779×10^3 km to standard form.
 (a) 677.79 km (b) 67.779 km (c) 67779 km (d) 6777.9 km
15. The exponential form of $\log_5^{25} = 2$ is:
 (a) $25^2 = 5$ (b) $2^5 = 25$ (c) $5^2 = 25$ (d) $25^2 = 2$
16. The logarithmic form of $10^3 = 1000$ is:
 (a) $\log_{10} 3 = 1000$ (b) $\log_3 10 = 1000$ (c) $\log_{10} 1000 = 3$ (d) $\log_3 1000 = 10$
17. If $\log_3 x = 5$, then $x =$
 (a) 10 (b) 32 (c) 25 (d) 64
18. If $\log_a b = c$, then:
 (a) $c^a = b$ (b) $b^c = a$ (c) $a^b = c$ (d) $a^c = b$
19. If $\log x = 2.3010$, then $x =$
 (a) 20 (b) 200 (c) 0.2 (d) 2000

Unit-2

LOGARITHMS
Short Questions

Exercise 2.1

1. Define scientific notation.

Ans: Scientific notation: A number in scientific notation is written as $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{Z}$. Here "a" is called the coefficient or base number.

2. Convert 78,000,000 to scientific notation.

Solution:
 $78,000,000 = 7.8 \times 10^7$
 Since we moved the decimal to the left, the exponent is positive.

3. Convert 0.0000000315 to scientific notation.

Solution:
 $0.0000000315 = 3.15 \times 10^{-8}$
 Since we moved the decimal to the right, the exponent is negative.

4. Convert 0.000567 to scientific notation.

Solution:
 0.000567
 $= 5.67 \times 10^{-4}$

5. Convert 734 to scientific notation.

Solution:
 734
 $= 7.34 \times 10^2$

6. Convert 0.33×10^3 to scientific notation.

Solution:
 0.33×10^3
 $= 3.3 \times 10^2$

$$= 3.3 \times 10^{3-1}$$

$$= 3.3 \times 10^2$$

7. Convert 3.47×10^6 to ordinary notation.

Solution:
 $3.47 \times 10^6 = 3,470,000$
 Since the exponent is positive 6, move the decimal point 6 places to right.

8. Convert 6.23×10^{-4} to ordinary notation.

Solution:
 $6.23 \times 10^{-4} = 0.000623$
 Since the exponent is negative 4, move the decimal point 4 places to left.

9. Convert 2.6×10^3 to ordinary notation.

Solution:
 2.6×10^3
 $= 2600$

10. Convert 8.794×10^{-4} to ordinary notation.

Solution:
 8.794×10^{-4}
 $= 0.0008794$

11. Convert 6×10^{-4} to ordinary notation.

Solution:
 6×10^{-4}
 $= 0.000006$

12. The speed of light is approximately 3×10^8 meters per second. Express it in standard form.

Solution:

$$= 3 \times 10^8 \text{ m/sec}$$
$$= 300,000,000 \text{ m/sec}$$

13. The diameter of Mars is 6.779×10^3 km. Express this number in standard form.

Solution:

$$= 6.779 \times 10^3 \text{ km}$$
$$= 6.779 \times 1000 \text{ km}$$
$$= 6779 \text{ km}$$

Exercise 2.2

14. What do you mean of logarithm.

Ans: Logarithm: A logarithm is based on two Greek words: logos and arithmos which means ratio or proportion.

15. Write general form of logarithm.

Ans: The general form of a logarithm is:
 $\log_b(x) = y$

Where:

b is the base,
 x is the result or the number whose logarithm is being taken,
 y is the exponent or the logarithm of x to the base b .

16. Express in logarithmic form: $2^8 = 256$

Solution:

$$2^8 = 256$$
$$\log 2^{256} = 8$$

17. Express in logarithmic form: $a^b = c$

Solution:

$$a^b = c$$
$$\log a^c = b$$

18. Express in exponential form:

$$\log_2 16 = 4$$

Solution:

$$\log_2 16 = 4$$
$$2^4 = 16$$

19. Express in exponential form:

$$\log_9 729 = 3$$

Solution:

$$\log_9 729 = 3$$
$$9^3 = 729$$

20. Find the value of x : $\log_5 1 = x$

Solution:

$$\log_5 1 = x$$
$$5^x = 1$$
$$5^x = 5^0$$
$$x = 0$$

21. Find the value of x : $\log_4 x = \frac{3}{2}$

Solution:

$$\log_4 x = \frac{3}{2}$$

$$4^{\frac{3}{2}} = x$$

$$x = (2^2)^{\frac{3}{2}}$$

$$x = 2^{2 \times \frac{3}{2}}$$

$$x = 2^3$$

$$x = 8$$

22. Find value of x : $\log 9^x = 0.5$

Solution:

$$\log 9^x = 0.5$$

$$9^{0.5} = x$$

$$9^{\frac{1}{2}} = x$$

$$x = \sqrt{9}$$

$$x = \sqrt{3^2}$$

$$x = 3$$

23. Express in exponential form:

$$\log_4 8 = x$$

Solution:

$$\log_4 8 = x$$

$$4^x = 8$$

24. Express in exponential form:

$$\log_4 1024 = 5$$

Solution:

$$\log_4 1024 = 5$$

$$4^5 = 1024$$

Exercise 2.3

25. Define common logarithm.

Ans: Common logarithm: The common logarithm is the logarithm with a base of 10. It is written as \log_{10} or simply as \log (when no base is specified, it is usually assumed to be base 10).

26. Define characteristic.

Ans: Characteristic: The characteristic is the integral part of the logarithm. It tells us how big or small the number is.

27. Define mantissa.

Ans: Mantissa: The mantissa is the decimal part of the logarithm. It is always positive.

28. Define antilogarithm.

Ans: Antilogarithm: If $\log_b(x) = y \Leftrightarrow b^y = x$ then the process of finding x is called antilogarithm of y .

29. Define natural logarithm.

Ans: Natural logarithm: The natural

logarithm is the logarithm with base e , where e is a mathematical constant approximately equal to 2.71828..... It is denoted as \ln .

30. Find characteristic of 234.7

Solution:

234.7

Characteristic = $3 - 1 = 2$

31. Find characteristic of 0.000049

Solution:

0.000049

Characteristic = $-(4 + 1) = \bar{5}$

32. Find logarithm of 579

Solution:

579

Let $x = 579$

Taking log on both side:

$\log x = \log 579$

$\log x = 2.7627$

$\log 579 = 2.7627$

33. Find logarithm of 0.047

Solution:

0.047

Let $x = 0.047$

Taking log on both side:

$\log x = \log 0.047$

$\log x = \bar{2}.6721$

$\log 0.047 = \bar{2}.6721$

34. Find logarithm of 1.982

Solution:

1.982

Let $x = 1.982$

Taking log on both side:

$\log x = \log 1.982$

$\log x = 0.2971$

$\log 1.982 = 0.2971$

35. Find logarithm of 0.0876

Solution:

0.0876

Let $x = 0.0876$

Taking log on both side:

$\log x = \log 0.0876$

$\log x = \bar{2}.9425$

$\log 0.0876 = \bar{2}.9425$

36. Find logarithm of 0.000354

Solution:

0.000354

Let $x = 0.000354$

Taking log on both side:

$\log x = \log 0.000354$

$\log x = \bar{4}.5490$

$\log 0.000354 = \bar{4}.5490$

37. Find the value of x : $\log x = -1.4567$

Solution:

$\log x = -1.4567$

Since mantissa is negative, so we make it positive by adding and subtracting 2

$\log x = -1.4567 + 2 - 2$

$\log x = \bar{2}.5433$

Taking antilog on both sides:

Anti log ($\log x$) = Anti log ($\bar{2}.5433$)

$x = 0.03493$

38. Find the value of x : $\log x = 1.192$

Solution:

$\log x = 1.192$

Taking Antilog on both side

Antilog ($\log x$) = Antilog (1.192)

$x = 15.56$

39. Find the value of x : $\log x = 4.3561$

Solution:

$\log x = 4.3561$

Taking Antilog on both side:

Antilog ($\log x$) = Antilog (4.3561)

$x = 22703$

40. Find the value of x : $\log x = 0.0065$

Solution:

$\log x = 0.0065$

Taking Antilog on both side:

Antilog ($\log x$) = Antilog (0.0065)

$x = 1.015$

41. Find the value of x : $\log x = 4.3561$

Solution:

$\log x = -2.0184$

$\log x = -2.0184 + 3 - 3$

$\log x = \bar{3}.9816$

Taking Antilog on both side

Antilog ($\log x$) = Antilog ($\bar{3}.9816$)

$x = 0.009585$

Unit-2

LOGARITHMS Long Questions

- The speed of light is approximately 3×10^8 meters per second. Express it in standard form.
- The circumference of the Earth at the equator is about 40075000 meters. Express this number in scientific notation.
- The diameter of Mars is 6.779×10^3 km. Express this number in standard form.
- The diameter of Earth is about 1.2756×10^4 km. Express this number in standard form.

5. Find the value of x : (i) $\log_4 64 = 3$ (ii) $\log_4 1 = x$
 6. Find the value of x : (i) $\log_4 8 = 1$ (ii) $\log_{16} x = -3$
 7. Find the value of x : (i) $\log_4 x = \frac{3}{2}$ (ii) $\log_4 1024 = x$
 8. Find the value of x in each case: (i) $\log_5 25 = x$ (ii) $\log_4 x = 6$
 9. Find the value of x : $\log x = 0.0065$ 10. Find the value of x : $\log x = 1.192$
 11. Find the value of x : $\log x = -3.434$ 12. Find the value of x : $\log x = -1.5726$
 13. Find the value of x : $\log x = 4.3561$ 14. Find the value of x : $\log x = -2.0184$
 15. Find the value of x : $\log 9^x = 0.5$ 16. Find the value of x : $\left(\frac{1}{9}\right)^{3x} = 27$
 17. Find the value of x : $\left(\frac{1}{32}\right)^{3x} = 64$

Unit-3

**SET AND FUNCTIONS
MCQs**

Q.1: Four options are given against each statement. Encircle the correct option.

- The set builder form of the set $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots\right\}$ is:
 - $\left\{x \mid x = \frac{1}{n}, n \in \mathbb{W}\right\}$
 - $\left\{x \mid x = \frac{1}{2n+1}, n \in \mathbb{W}\right\}$
 - $\left\{x \mid x = \frac{1}{n+1}, n \in \mathbb{W}\right\}$
 - $\left\{x \mid x = 2n+1, n \in \mathbb{W}\right\}$
- If $A = \{\}$, then $P(A)$ is:
 - $\{\}$
 - $\{\{\}\}$
 - $\{\{\}\}$
 - \emptyset
- If $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $U - (A \cap B)$ is:
 - $\{1, 2, 4, 5\}$
 - $\{2, 3\}$
 - $\{1, 3, 4, 5\}$
 - $\{1, 2, 3\}$
- If A and B are overlapping sets, the $n(A - B)$ is equal to
 - $n(A)$
 - $n(B)$
 - $A \cap B$
 - $n(A) - n(A \cap B)$
- If $A \subseteq B$ and $B - A \neq \emptyset$, then $n(B - A)$ is equal to
 - 0
 - $n(B)$
 - $n(A)$
 - $n(B) - n(A)$
- If $n(A \cup B) = 50$, $n(A) = 30$ and $n(B) = 25$, then $n(A \cap B) =$
 - 23
 - 15
 - 9
 - 40
- Example of equal set:
 - $\{a, b, c\}, \{1, 2, 3\}$
 - $\{1, 2, 3\}, \{2, 1, 3\}$
 - $\{\alpha, \beta, \gamma\}, \{a, b, c\}$
 - none of these
- Symbol used for equivalent set A and B :
 - $A = B$
 - $A \equiv B$
 - $A - B$
 - $A \subseteq B$
- element of power set $C = \{a, b, c, d\}$:
 - 8
 - 16
 - 24
 - 24
- If $D = \{a\}$ then $P(D) =$:
 - $\{a\}$
 - $\emptyset, \{a\}$
 - $\{\emptyset, \{a\}\}$
 - none of these
- $\{x \mid x \in A \vee x \in B\}$ represent as:
 - $A \cup B$
 - $A \cap B$
 - $A - B$
 - \emptyset
- Georg cantor was _____ mathematician.
 - Japani
 - German
 - Greek
 - Egyptain
- Set $A = \{a, e, i, o, u\}$ written in form:
 - descriptive
 - tabular
 - builder
 - none of these
- Set $\{2, 3, 5, 7, 11, \dots\}$ is called _____ member:
 - natural
 - whole
 - odd
 - prime

15. QUQ' = _____
 (a) ϕ (b) R (c) Q (d) Q'
16. The set $\{0\}$ is:
 (a) singleton (b) empty (c) null (d) prime
17. The set with no element is called _____ set:
 (a) singleton (b) empty (c) power (d) prime
18. Symbol used for empty set:
 (a) $\{\phi\}$ (b) ϕ (c) $\{\{\}\}$ (d) none of these
19. If A subset of Set B then written as symbolically:
 (a) $A \subseteq B$ (b) $A \supseteq B$ (c) $A \in B$ (d) $A \notin B$
20. $A \cap B =$ _____
 (a) $\{x|x \in A \vee x \in B\}$ (b) $\{x|x \in A \wedge x \in B\}$ (c) $\{x|x \in A \wedge x \notin B\}$ (d) $\{x|x \notin A \wedge x \in B\}$
21. $(A \cup B)'$ = _____
 (a) $A' \cap B'$ (b) $A' \cup B'$ (c) $A' \cap B$ (d) $A' \cup B$
22. $A \cap B = B \cap A$ is called _____ Property of intersection:
 (a) associative (b) commutative (c) distributive (d) none of these
23. $A' =$ _____
 (a) $U - A$ (b) $A - U$ (c) U (d) ϕ
24. $A' =$ _____
 (a) $\{x|x \in U \wedge x \notin A\}$ (b) $\{x|x \in A \wedge x \in U\}$ (c) $\{x|x \in U \wedge x \in A\}$ (d) none of these
25. In ordered pair (x, y) x is called:
 (a) Abscissa (b) ordinate (c) both (d) none of these

Unit-3

SET AND FUNCTIONS
Short Questions

Exercise 3.1

1. **Ans:** How many methods to describe a set.
 There are three different ways to describing a set.
 (i) The Descriptive form
 (ii) The Tabular form
 (iii) The set-builder method
2. **Ans:** Define singleton set.
 Singleton set: A set with only one element is called a singleton set.
3. **Ans:** Define empty set.
 Empty set: The set with no elements (zero number of elements) is called an empty set, null set, or Void set. The empty set is denoted by the symbol ϕ or $\{\}$.
4. **Ans:** Define subset.
 Subset: If every element of a set A is an element of set B, then A is a subset of B.

- Symbolically written as $A \subseteq B$.
Example: $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5, 6\}$
5. **Ans:** Define proper subset.
 Proper subset: If A is a subset of B and B contains at least one element that is not an element of A, then A is said to be a proper subset of B. Symbolically write as: $A \subset B$.
6. **Ans:** Define universal set.
 Universal set: The set that contains all objects or elements under consideration is called the universal set or the universe of discourse. It is denoted by U.
7. **Ans:** Define Power set.
 Power set: The power set of a set A denoted by $P(A)$ is the set containing all the possible subsets of A.
Example: If $A = \{a\}$ then $P(A) = \{\phi, \{a\}\}$

8. Define Union of two sets.

Ans: Union of two sets: The union of two sets A and B, denoted by $A \cup B$, is the set of all elements which belong to A or B.

Symbolically; $A \cup B = \{x | x \in A \vee x \in B\}$

Example: If $A = \{1, 2, 3\}$ $B = \{2, 3, 4, 5\}$ then $A \cup B = \{1, 2, 3, 4, 5\}$

9. Define intersection of two sets.

Ans: Intersection of two sets: The intersection of two sets A and B, denoted by $A \cap B$ is the set of all elements that belong to both A and B.

Symbolically: $A \cap B = \{x | x \in A \wedge x \in B\}$.

Example: If $A = \{1, 2, 3\}$ $B = \{2, 3, 4, 5\}$ then $A \cap B = \{2, 3\}$

10. Define difference of two sets.

Ans: Difference of two sets: The difference between the sets A and B denoted by $A - B$ or $A \setminus B$, consists of all the element that belong to A but do not belong to B. Symbolically, $A - B = \{x | x \in A \wedge x \notin B\}$.

Example:

If $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7, 8, 9, 10\}$
 $A - B = \{1, 2, 3\}$ and $B - A = \{6, 7, 8, 9, 10\}$.

11. Define disjoint set.

Ans: Disjoint set: If the intersection of two sets is the empty set, the sets are said to be disjoint.

Example:

If $S_1 =$ The set of odd natural numbers
 $S_2 =$ The set of even natural numbers
 S_1 and S_2 are disjoint sets.

12. Write the set in builder notation:

$\{5, 10, 15, \dots, 100\}$

Solution:

$\{5, 10, 15, \dots, 100\}$
 $\{x | x = 5n, n \in \mathbb{N} \wedge 1 \leq n \leq 20\}$

13. Write the set in builder notation:

Set of all integers between -100 and 1000

Solution:

Set of all integers between -100 and 1000
 $\{x | x \in \mathbb{Z} \wedge -100 < x < 1000\}$

14. Write the set in builder notation:

$\{6, 12, 18, \dots, 120\}$

Solution:

$\{6, 12, 18, \dots, 120\}$
 $\{x | x = 6n, n \in \mathbb{N} \wedge 1 \leq n \leq 20\}$

15. Write the set in builder notation:

$\{1, 3, 9, 27, 81, \dots\}$

Solution:

$\{1, 3, 9, 27, 81, \dots\}$
 $\{x | x = 3^n, n \in \mathbb{W}\}$

16. Write the sets in tabular forms:

$\{x | x \text{ is a multiple of } 3 \wedge x \leq 36\}$

Solution:

$\{x | x \text{ is a multiple of } 3 \wedge x \leq 36\}$
 $\{3, 6, 9, \dots, 36\}$

17. Write the sets in tabular forms:

$\{x | x \in \mathbb{R} \wedge 2x + 1 = 0\}$

Solution:

$\{x | x \in \mathbb{R} \wedge 2x + 1 = 0\}$
 $\left\{-\frac{1}{2}\right\}$

18. Write the sets in tabular forms:

$\{x | x = 2^n, n \in \mathbb{N} \wedge n < 8\}$

Solution:

$\{x | x = 2^n, n \in \mathbb{N} \wedge n < 8\}$
 $\{2, 4, 8, 16, 32, 64, 128\}$

19. Write the sets in tabular forms:

$\{x | x \in \mathbb{N} \wedge x + 4 = 0\}$

Solution:

$\{x | x \in \mathbb{N} \wedge x + 4 = 0\}$
 $\{\}$

20. Write two proper subsets.

Solution:

Two proper subset:

$\{a, b, c\}$
 $\{a\}, \{b\}$

21. Is there any set which has no proper subset? If so, name that set.

Solution:

Yes, empty set $\{\}$ has no proper subset.

22. What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?

Solution:

$\{a, b\}$ is a set containing two elements a and b while $\{\{a, b\}\}$ is a set containing one element $\{a, b\}$.

23. Write down the power set: $\{9, 11\}$

Solution:

$\{9, 11\}$
 $\{\phi, \{9\}, \{11\}, \{9, 11\}\}$

24. Write down the power set: $\{+, -, \times, \div\}$

Solution:

$\{+, -, \times, \div\}$
 $\{\phi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+, -\}, \{+, \times\}, \{+, \div\}, \{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\}, \{+, \times, \div\}, \{-, \times, \div\}, \{+, -, \times, \div\}\}$

Exercise 3.2

25. Define overlapping set.

Ans: Overlapping set: If the intersection of two sets is non-empty but neither is a

subset of the other, the sets are called overlapping sets.

Example: $L = \{2, 3, 4, 5, 6\}$ and

$M = \{5, 6, 7, 8, 9, 10\}$ then L and M are overlapping sets.

26. Define complement of a set.

Ans: Complement of a set: The complement of a set A, denoted by A' or A^c relative to the universal set U is the set of all elements of U, which do not belong to A. Symbolically:

$$A' = \{x | x \in U \wedge x \notin A\}$$

27. Write De Morgan's Laws.

Ans: De Morgan's Laws:

(i). $(A \cup B)' = A' \cap B'$

(ii). $(A \cap B)' = A' \cup B'$

28. Consider the universal set

$$U = \{x | x \text{ is multiple of } 2 \wedge 0 < x \leq 30\}$$

$$A = \{x | x \text{ is a multiple of } 6\} \text{ and}$$

$$B = \{x | x \text{ is a multiple of } 8\}$$

- (i) List all elements of sets A and B in tabular form (ii) Find $A \cap B$
(iii) Draw a Venn diagram

Solution:

(a) List all elements of sets A and B in tabular form

$$A = \{6, 12, 18, 24, 30\}$$

$$B = \{8, 16, 24\}$$

(b) Find $A \cap B$

$$= \{6, 12, 18, 24, 30\} \cap \{8, 16, 24\}$$

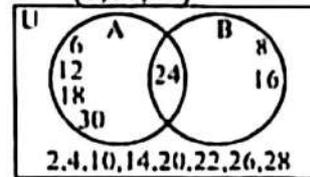
$$= \{24\}$$

(c) Draw a Venn diagram

$$U = \{2, 4, 6, 8, \dots, 28, 30\}$$

$$A = \{6, 12, 18, 24, 30\}$$

$$B = \{8, 16, 24\}$$



29. If $U = \{1, 2, 3, \dots, 20\}$ and

$$A = \{1, 3, 5, \dots, 19\}, \text{ verify } A \cup A' = U$$

Solution:

$$A \cup A' = U$$

$$\text{L.H.S} = A \cup A'$$

$$A' = U - A$$

$$= \{1, 2, 3, \dots, 20\} - \{1, 3, 5, 7, 9, 11, \dots, 19\}$$

$$= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

$$A \cup A' = \{1, 3, 5, 7, 9, 11, \dots, 19\} \cup \{2, 4, 6, 8, \dots, 20\}$$

$$A \cup A' = \{1, 2, 3, 4, 5, \dots, 20\}$$

$$A \cup A' = U$$

30. What do you know about Venn diagram

Ans:

British mathematician John Venn introduced rectangle for a universal set U and its subsets A and B as closed figures. Venn diagrams are very useful in depicting visually the basic concepts of sets and relationships between sets.

Unit-3

SET AND FUNCTIONS

Long Questions

- Let $U = \{a, b, c, d, e, f, g, h, i, j\}$ $A = \{a, b, c, d, g, h, i\}$ $B = \{c, d, e, f, j\}$
Verify De Morgan's laws for these sets, Draw Venn diagram
- If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $C = \{5, 6, 7, 9, 10\}$ then verify: $A \cup (B \cap C) = (A \cup B) \cap C$
- If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $C = \{5, 6, 7, 9, 10\}$ then verify
 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- Verify the commutative properties of union and intersection.
 $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 6, 8, 10\}$
- Let $U = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{a, b, c, d, g, h\}$, $B = \{c, d, e, f, j\}$
Verify De Morgan's laws for these sets, Draw Venn diagram
- Verify Associative of union $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $C = \{5, 6, 7, 9, 10\}$.
- Verify Distributive of intersection over union $A = \phi$, $B = \{0\}$, $C = \{0, 1, 2\}$
- In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each student likes to play at least one of the two games. How many students like to play both games?
- There are 98 secondary school students in a sports club. 58 students the swimming club, and 50 join the tug-of-war club. How many students participated in both games?
- In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak both Urdu and English, 30 can speak both English and Punjabi and 10 can speak both Urdu and Punjabi. How many can speak all three languages?

FACTORIZATION AND ALGEBRAIC MANIPULATION MCQs

1. Choose the correct option.

1. The factorization of $12x + 36$ is:
 (a) $12(x + 3)$ (b) $12(3x)$ (c) $12(3x + 1)$ (d) $x(12 + 36x)$
2. The factors of $4x^2 - 12x + 9$ are:
 (a) $(2x + 3)^2$ (b) $(2x - 3)^2$
 (c) $(2x - 3)(2x + 3)$ (d) $(2 + 3x)(2 - 3x)^2$
3. The HCF of a^3b^3 and ab^2 is:
 (a) a^3b^3 (b) ab^2 (c) a^4b^3 (d) a^2b
4. The LCM of $16x^2$, $4x$ and $30xy$ is:
 (a) $480x^2y$ (b) $240xy$ (c) $240x^2y$ (d) $120x^4y$
5. Product of LCM and HCF = _____ of two polynomials.
 (a) sum (b) difference (c) product (d) quotient
6. L.C.M = _____
 (a) $\frac{P(x) \times q(x)}{H.C.F}$ (b) $\frac{H.C.F \times P(x)}{q(x)}$ (c) $P(x) \times q(x) \times H.C.F$ (d) none of these
7. The LCM of $(a - b)^3$ and $(a - b)^4$ is:
 (a) $(a - b)^2$ (b) $(a - b)^3$ (c) $(a - b)^4$ (d) $(a - b)^6$
8. Factorization of $x^3 + 3x^2 + 3x + 1$ is:
 (a) $(x + 1)^3$ (b) $(x - 1)^3$
 (c) $(x + 1)(x^2 + x + 1)$ (d) $(x - 1)(x^2 - x + 1)$
9. Cubic polynomial has degree:
 (a) 1 (b) 2 (c) 3 (d) 4
10. One of the factors of $x^2 - 27$ is:
 (a) $x - 3$ (b) $x + 3$ (c) $x^2 - 3x + 9$ (d) Both a and c
11. Common factor of $2x - 6$ is:
 (a) 2 (b) 3 (c) x (d) none of these
12. Factors of $x^2 - 3x - 10$ are:
 (a) $(x + 2)(x + 5)$ (b) $(x + 2)(x - 5)$ (c) $(x - 2)(x + 5)$ (d) $(x - 2)(x - 5)$
13. Which expression type of quadratic expression?
 (a) $ax + b$ (b) $ax^2 + bx + c$ (c) $ax^3 + bx^2 + c$ (d) none of these
14. Which pair is suitable for factorization of $x^2 - 11x + 24$?
 (a) +3, +8 (b) -3, -8 (c) -3, +8 (d) +3, -8
15. An expression having degree 2 is called _____ expression:
 (a) cubic (b) linear (c) quadratic (d) by quadrative
16. Factors of $3x^2 - 4x - 4$ are:
 (a) $(3x + 2)(x - 2)$ (b) $(3x - 2)(x + 2)$ (c) $(3x + 2)(x + 2)$ (d) none of these
17. Which of the following is a prime number?
 (a) 9 (b) 15 (c) 13 (d) 21
18. $a^2 - 2ab + b^2$ is equal to:
 (a) $(a + b)^2$ (b) $(a - b)^2$ (c) $a^2 - b^2$ (d) $a^2 + b^2$
19. $a^3 + b^3 + 3ab(a + b)$ is equal to:
 (a) $(a + b)^3$ (b) $(a - b)^3$ (c) $a^3 - b^3$ (d) $a^3 + b^3$
20. Factors of $8x^3 + 27$
 (a) $(2x + 3)(4x^2 + 6x + 9)$ (b) $(2x + 3)(4x^2 - 6x + 9)$
 (c) $(2x - 3)(4x^2 + 6x + 9)$ (d) $(2x - 3)(4x^2 - 6x + 9)$
21. How many method to solve H.C.F.
 (a) 1 (b) 2 (c) 3 (d) 4
22. H.C.F of $6x^2y$, $9xy^2$
 (a) $3x^2y$ (b) $3xy$ (c) $6x^2y$ (d) $54x^2y^3$
23. Square root of $a^2 - 2a + 1$
 (a) $\pm(a - 1)$ (b) $\pm(a + 1)$ (c) $a - 1$ (d) $a + 1$

24. In $2x^2 + 7x + 3$ one factor is $2x + 1$ then other factor.
 (a) $(x + 2)$ (b) $(x + 1)$ (c) $(x + 3)$ (d) $(x + 4)$
25. An expression with two term is called:
 (a) monomial (b) Binomial (c) Trinomial (d) none of these

Unit-4

FACTORIZATION AND ALGEBRAIC MANIPULATION
Short Questions

Exercise 4.1

1. Define Quadratic Equation.

Ans: Quadratic Equation: An expression having degree 2 is called a quadratic expression. $ax^2 + bx + c = 0$

2. Define factorization.

Ans: Factorization: The process of expressing an algebraic expression in terms of its factors is called factorization.

3. Define common factorization.

Ans: Common factorization: In algebra, a common factor is an expression that divides two or more expressions exactly.

4. Factorize: $x^2 + 9x + 14$

Solution:

$$\begin{aligned} &x^2 + 9x + 14 \\ &= x^2 + 2x + 7x + 14 \\ &= x(x + 2) + 7(x + 2) \\ &= (x + 2)(x + 7) \end{aligned}$$

5. Factorize: $x^2 - 11x + 24$

Solution:

$$\begin{aligned} &x^2 - 11x + 24 \\ &= x^2 - 8x - 3x + 24 = x(x - 8) - 3(x - 8) \\ &= (x - 8)(x - 3) \end{aligned}$$

6. Factorize by identifying common factors $-12x^2 - 3x$

Solution:

$$\begin{aligned} &-12x^2 - 3x \\ &= -3x(4x + 1) \end{aligned}$$

7. Factorize by identifying common factors: $xy - 3x^2 + 2x$

Solution:

$$\begin{aligned} &xy - 3x^2 + 2x \\ &= x(y - 3x + 2) \end{aligned}$$

8. Factorize: $x^2 - x - 56$

Solution:

$$\begin{aligned} &x^2 - x - 56 \\ &= x^2 - 8x + 7x - 56 = x(x - 8) + 7(x - 8) \\ &= (x - 8)(x + 7) \end{aligned}$$

9. Factorize: $x^2 - 10x - 24$

Solution:

$$\begin{aligned} &x^2 - 10x - 24 \\ &= x^2 - 12x + 2x - 24 = x(x - 12) + 2(x - 12) \\ &= (x - 12)(x + 2) \end{aligned}$$

10. Factorize: $y^2 + 13y + 36$

Solution:

$$\begin{aligned} &y^2 + 13y + 36 \\ &= y^2 + 9y + 4y + 36 \\ &= y(y + 9) + 4(y + 9) \\ &= (y + 9)(y + 4) \end{aligned}$$

11. Factorize: $x^2 - x - 2$

Solution:

$$\begin{aligned} &x^2 - x - 2 \\ &= x^2 - 2x + x - 2 \\ &= x(x - 2) + 1(x - 2) \\ &= (x - 2)(x + 1) \end{aligned}$$

12. Factorize: $2x^2 + 7x + 3$

Solution:

$$\begin{aligned} &2x^2 + 7x + 3 \\ &= 2x^2 + 6x + x + 3 \\ &= 2x(x + 3) + 1(x + 3) \\ &= (x + 3)(2x + 1) \end{aligned}$$

13. Factorize: $3y^2 - 11y + 6$

Solution:

$$\begin{aligned} &3y^2 - 11y + 6 \\ &= 3y^2 - 9y - 2y + 6 \\ &= 3y(y - 3) - 2(y - 3) \\ &= (y - 3)(3y - 2) \end{aligned}$$

14. Factorize: $4z^2 - 11z + 6$

Solution:

$$\begin{aligned} &4z^2 - 11z + 6 \\ &= 4z^2 - 8z - 3z + 6 \\ &= 4z(z - 2) - 3(z - 2) \\ &= (z - 2)(4z - 3) \end{aligned}$$

15. Factorize: $6 + 7x - 3x^2$

Solution:

$$\begin{aligned} &6 + 7x - 3x^2 \\ &= 6 + 9x - 2x - 3x^2 \end{aligned}$$

$$\begin{aligned}
 &= 3(2+3x) - x(2+3x) \\
 &= (2+3x)(3-x) \\
 \text{16. Factorize: } &4x^3 + 18x^2 - 12x
 \end{aligned}$$

Solution:

$$\begin{aligned}
 &4x^3 + 18x^2 - 12x \\
 &= 2x(2x^2 + 9x - 6)
 \end{aligned}$$

17. Factorize: $-x^2 - 23x - 60$

Solution:

$$\begin{aligned}
 &-x^2 - 23x - 60 \\
 &= -(x^2 + 23x + 6)
 \end{aligned}$$

$$\begin{aligned}
 &= -(x^2 + 20x + 3x + 60) \\
 &= -[x(x+20) + 3(x+20)] \\
 &= -(x+3)(x+20)
 \end{aligned}$$

18. Factorize: $y^2 + 4y - 12$

Solution:

$$\begin{aligned}
 &y^2 + 4y - 12 \\
 &= y^2 + 6y - 2y - 12 \\
 &= y(y+6) - 2(y+6) \\
 &= (y+6)(y-2)
 \end{aligned}$$

Exercise 4.2

19. Define H.C.F.

Ans:

Highest Common Factor (HCF)

The HCF of algebraic expressions refers to the greatest algebraic expression that divides two or more algebraic expressions without leaving a remainder.

20. Define L.C.M

Ans:

Least Common Multiple (LCM)

The LCM of two or more algebraic expressions is the smallest expression that is divisible by each of the given expressions.

21. How many method to solve H.C.F.

Ans:

We can find HCF of given expressions by the following two methods:

- (a) By factorization
- (b) By division

22. What relationship between LCM and HCF.

Ans:

The relationship between LCM and HCF can be expressed as follows:

$$\text{LCM} \times \text{HCF} = p(x) \times q(x)$$

Where, $p(x)$ = First polynomial
 $q(x)$ = Second polynomial

23. Factorize: $x^4 + x^2 + 25$

Solution:

$$\begin{aligned}
 &x^4 + x^2 + 25 \\
 &= x^4 + 25 + x^2 \\
 &= (x^2)^2 + (5)^2 + 2(x^2)(5) - 2(x^2)(5) + x^2 \\
 &= (x^2 + 5)^2 - 10x^2 + x^2 \\
 &= (x^2 + 5)^2 - 9x^2 \\
 &= (x^2 + 5)^2 - (3x)^2 \\
 &= (x^2 + 5 - 3x)(x^2 + 5 + 3x) \\
 &= (x^2 - 3x + 5)(x^2 + 3x + 5)
 \end{aligned}$$

24. Factorize: $a^4 + 64$

Solution:

$$\begin{aligned}
 &a^4 + 64 \\
 &= (a^2)^2 + (8)^2 \\
 &= (a^2)^2 + (8)^2 + 2(a^2)(8) - 2(a^2)(8) \\
 &= (a^2 + 8)^2 - 16a^2 \\
 &= (a^2 + 8)^2 - (4a)^2 \\
 &= (a^2 + 8 - 4a)(a^2 + 8 + 4a) \\
 &= (a^2 - 4a + 8)(a^2 + 4a + 8)
 \end{aligned}$$

25. Factorize: $x^4 - 30x^2y^2 + 9y^4$

Solution:

$$\begin{aligned}
 &x^4 - 30x^2y^2 + 9y^4 \\
 &= (x^2)^2 + (3y^2)^2 + 2(x^2)(3y^2) - 2(x^2)(3y^2) - 30x^2y^2 \\
 &= (x^2 + 3y^2)^2 - 6x^2y^2 - 30x^2y^2 \\
 &= (x^2 + 3y^2)^2 - 36x^2y^2 \\
 &= (x^2 + 3y^2)^2 - (6xy)^2 \\
 &= [(x^2 + 3y^2) + 6xy][(x^2 + 3y^2) - 6xy] \\
 &= (x^2 + 3y^2 + 6xy)(x^2 + 3y^2 - 6xy) \\
 &= (x^2 + 6xy + 3y^2)(x^2 - 6xy + 3y^2)
 \end{aligned}$$

26. Factorize: $x^4 + 2x^2 + 9$

Solution:

$$\begin{aligned}
 &x^4 + 2x^2 + 9 \\
 &= (x^2)^2 + (3)^2 + 2(x^2)(3) - 2(x^2)(3) + 2x^2 \\
 &= (x^2 + 3)^2 - 6x^2 + 2x^2 \\
 &= (x^2 + 3)^2 - 4x^2 \\
 &= (x^2 + 3)^2 - (2x)^2 \\
 &= [(x^2 + 3) + 2x][(x^2 + 3) - 2x] \\
 &= (x^2 + 2x + 3)(x^2 - 2x + 3)
 \end{aligned}$$

27. Factorize: $8x^3 + 60x^2 + 150x + 125$

Solution:

$$\begin{aligned}
 &8x^3 + 60x^2 + 150x + 125 \\
 &= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3 \\
 &= (2x + 5)^3 \\
 &= (2x + 5)(2x + 5)(2x + 5)
 \end{aligned}$$

28. Factorize: $x^3 + 18x^2y + 108xy^2 + 216y^3$

Solution:

$$\begin{aligned}
 &x^3 + 18x^2y + 108xy^2 + 216y^3 \\
 &= (x)^3 + 3(x)^2(6y) + 3(x)(6y)^2 + (6y)^3 \\
 &= (x + 6y)^3
 \end{aligned}$$

26. Factorize: $8x^3 - 125y^3 + 150xy^2 - 60x^2y$

Solution:

$$\begin{aligned}
 &8x^3 - 60x^2y + 150xy^2 - 125y^3 \\
 &= (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3 \\
 &= (2x - 5y)^3
 \end{aligned}$$

29. Factorize: $x^3y^3 - 8$

Solution:

$$\begin{aligned}
 &x^3y^3 - 8 \\
 &= (xy)^3 - (2)^3 \\
 &= (xy - 2)[(xy)^2 + (xy)(2) + (2)^2] \\
 &= (xy - 2)(x^2y^2 + 2xy + 4)
 \end{aligned}$$

30. Factorize: $343x^3 + 216$

Solution:

$$\begin{aligned} & 343x^3 + 216 \\ &= (7x)^3 + (6)^3 \\ &= (7x+6)[(7x)^2 - (7x)(6) + (6)^2] \\ &= (7x+6)(49x^2 - 42x + 36) \end{aligned}$$

31. Factorize: $27 - 512y^3$

Solution:

$$= (3)^3 - (8y)^3$$

33. Find the HCF of $6x^2y, 9xy^2$

Solution:

$$\begin{aligned} \text{Prime Factor of } 6x^2y &= 2 \times 3 \times x \times x \times y \\ \text{Prime Factor of } 9xy^2 &= 3 \times 3 \times x \times y \times y \\ \text{Common Factor} &= 3 \times x \times y \\ \text{HCF} &= 3xy \end{aligned}$$

34. Find the HCF by factorization method

$$x^3 - 27, x^2 + 6x - 27, x^2 - 9$$

Solution:

$$\begin{aligned} \text{Prime Factor of } x^3 - 27 &= (x)^3 - (3)^3 \\ &= (x-3)[(x)^2 + (3)(x) + (3)^2] \\ &= (x-3)(x^2 + 3x + 9) \end{aligned}$$

$$\begin{aligned} \text{Prime Factor of } x^2 + 6x - 27 &= x^2 + 9x - 3x - 27 \\ &= x(x+9) - 3(x+9) \\ &= (x+9)(x-3) \end{aligned}$$

$$\begin{aligned} \text{Prime Factor of } x^2 - 9 &= (x)^2 - (3)^2 \\ &= (x-3)(x+3) \end{aligned}$$

$$\text{Common Factor} = (x-3)$$

$$\text{Hence, HCF} = x-3$$

35. Find HCF by factorization method.

$$t^2 - 3t - 4, t^2 + 5t + 4, t^2 - 1$$

Solution:

$$\begin{aligned} \text{Prime factor of } t^2 - 3t - 4 &= t^2 - 4t + t - 4 \\ &= t(t-4) + 1(t-4) \\ &= (t-4)(t+1) \end{aligned}$$

$$\begin{aligned} \text{Prime factor of } t^2 + 5t + 4 &= t^2 + 4t + t + 4 \\ &= t(t+4) + 1(t+4) \\ &= (t+4)(t+1) \end{aligned}$$

$$\begin{aligned} \text{Prime factor of } t^2 - 1 &= (t)^2 - (1)^2 \\ &= (t+1)(t-1) \end{aligned}$$

$$\text{Common factor} = (t+1)$$

$$\text{HCF} = (t+1)$$

36. Find HCF by factorization method.

$$21x^2y, 35xy^2$$

Solution:

$$\text{Prime factor of } 21x^2y = 3 \times 7 \times x \times x \times y$$

$$\text{Prime factor of } 35xy^2 = 5 \times 7 \times x \times y \times y$$

$$\text{Common factor} = 7 \times x \times y$$

$$\text{HCF} = 7xy$$

$$\begin{aligned} &= (3-8y)[(3)^2 + (3)(8y) + (8y)^2] \\ &= (3-8y)(9+24y+64y^2) \end{aligned}$$

32. Factorize: $x^3 + 64y^3$

Solution:

$$\begin{aligned} & x^3 + 64y^3 \\ &= (x)^3 + (4y)^3 \\ &= (x+4y)[(x)^2 - (x)(4y) + (4y)^2] \\ &= (x+4y)(x^2 - 4xy + 16y^2) \end{aligned}$$

Exercise 4.3

37. Find the LCM of $4x^2y, 8x^3y^2$.

Solution:

$$\begin{aligned} \text{Prime Factor of } 4x^2y &= 2 \times 2 \times x \times x \times y \\ \text{Prime Factor of } 8x^3y^2 &= 2 \times 2 \times 2 \times x \times x \times x \times y \times y \\ \text{Common factors} &= 2 \times 2 \times x \times x \times y = 4x^2y \\ \text{Non-common factors} &= 2 \times x \times y = 2xy \\ \text{LCM} &= \text{C.F} \times \text{N.C.F} \\ &= 4x^2y \times 2xy \\ &= 8x^3y^2 \end{aligned}$$

38. Find LCM by using prime factorization method. $2a^2b, 4ab^2, 6ab$

Solution:

$$\begin{aligned} \text{Prime factor of } 2a^2b &= 2 \times a \times a \times b \\ \text{Prime factor of } 4ab^2 &= 2 \times 2 \times a \times b \times b \\ \text{Prime factor of } 6ab &= 2 \times 3 \times a \times b \\ \text{Common factor} &= 2 \times a \times b = 2ab \\ \text{Non-common factor} &= 2 \times 3 \times a \times b = 6ab \\ \text{LCM} &= \text{C.F} \times \text{N.C.F} \\ \text{LCM} &= 2ab \times 6ab \\ &= 12a^2b^2 \end{aligned}$$

39. Find LCM by using prime factorization method:

$$a^2 - 4a + 4, a^2 - 2a$$

Solution:

$$\begin{aligned} \text{Prime factor of } a^2 - 4a + 4 &= (a)^2 - 2(a)(2) + (2)^2 \\ &= (a-2)^2 \\ &= (a-2)(a-2) \\ \text{Prime factor of } a^2 - 2a &= a(a-2) \\ \text{Common factor} &= (a-2) \\ \text{Non-common factor} &= a(a-2) \\ \text{LCM} &= \text{C.F} \times \text{N.C.F} \\ \text{LCM} &= (a-2) \times a(a-2) \\ &= a(a-2)^2 \end{aligned}$$

40. Find HCF by prime factorization method:

$$4x^3 + 12x^2, 8x^2 + 16x$$

Solution:

$$4x^3 + 12x^2, 8x^2 + 16x$$

$$\begin{aligned} \text{Prime factor of } 4x^3 + 12x^2 &= 4x^2(x+3) \\ &= 2 \times 2 \times x \times x(x+3) \end{aligned}$$

9. The equation $2x - y = 4$ is equivalent to the inequality:
 (a) $2x - y > 4$ (b) $2x - y \leq 4$ (c) $2x - y \geq 4$ (d) None of these
10. If $x - y \leq 4$, then the region of solution is:
 (a) Above the line (b) Below the line (c) On the line (d) None of these
11. Which of the following is a linear equation in one variable?
 (a) $2x + 3y = 7$ (b) $5x - 4 = 0$ (c) $x^2 + 3x = 8$ (d) $xy + 2 = 5$
12. In the inequality $4x + 2 > 10$, what is the solution for x ?
 (a) $x > 2$ (b) $x < 2$ (c) $x > 1$ (d) $x < 1$
13. What is the solution of $2(x - 3) = 4$?
 (a) $x = 2$ (b) $x = 5$ (c) $x = 3$ (d) $x = 1$
14. The inequality $x - 5 \geq 3$ means:
 (a) x is greater than 3 (b) x is greater than or equal to 3
 (c) x is greater than or equal to 8 (d) x is less than 3
15. The equation $x + 2y = 6$ represents a:
 (a) quadratic equation (b) linear equation
 (c) exponential equation (d) none of these
16. The associated equation of the inequality $x + 2y \leq 8$ is:
 (a) $x + 2y = 8$ (b) $x + 2y > 8$ (c) $x + 2y < 8$ (d) $x + 2y \geq 8$
17. The solution of the system $x - y = 2$ and $x + y = 6$ is:
 (a) (4, 2) (b) (2, 4) (c) (3, 3) (d) (6, 0)
18. A vertical line divides the plane into:
 (a) Two half-planes (b) Four quadrants (c) Three sections (d) Infinite parts
19. The equation $x = 2$ represents a:
 (a) Horizontal line (b) Vertical line (c) Diagonal line (d) None of these

Unit-5

LINEAR EQUATIONS AND INEQUALITIES

Short Questions

Exercise 5.1

1. What is a linear equation in one variable?

Ans: **Linear equation:** A linear equation in one variable is an equation of the form $ax + b = 0$, where a and b are real number, and x is the variable.
 $ax + b = 0, \quad a \neq 0$

2. Define linear inequality.

Ans: **Linear inequality:** A linear inequality is a mathematical statement that relates linear expressions using inequality symbols ($>$, $<$, \geq , \leq).
 $ax + b < 0$

3. What is the associated equation of the inequality $2x + y \leq 8$?

Ans: The associated equation is $2x + y = 8$.

4. What is solution of equation $5x - 10 = 20$?

Solution:
 $5x - 10 = 20$
 $5x = 20 + 10$

$$5x = 30$$

$$x = \frac{30}{5}$$

$$x = 6$$

5. What is the solution of the inequality

$$3x - 4 \leq 8?$$

Solution:

$$3x - 4 \leq 8$$

$$3x \leq 8 + 4$$

$$3x \leq 12$$

$$x \leq \frac{12}{3}$$

$$x \leq 4$$

$$(x | x \leq 4)$$

6. Solve the system of equations $x + y = 6$ and $x - y = 2$.

Solution:

$$x + y = 6 \quad \dots\dots(i)$$

$$x - y = 2 \quad \dots\dots(ii)$$

Adding equation (i) and (ii)

$$x + y = 6$$

$$x - y = 2$$

$$2x = 8$$

$$x = \frac{8}{2}$$

$$x = 4$$

Putting the value of x in eq (i).

$$x + y = 6$$

$$4 + y = 6$$

$$y = 6 - 4$$

$$y = 2$$

7. Solve for x : $2(x - 1) = 3x + 2$.

Solution:

$$2(x - 1) = 3x + 2$$

$$2x - 2 = 3x + 2$$

$$2x - 3x = 2 + 2$$

$$-x = 4$$

$$x = -4$$

8. Solve for x : $4x + 3 = 19$.

Solution:

$$4x + 3 = 19$$

$$4x = 19 - 3$$

$$4x = 16$$

$$x = \frac{16}{4}$$

$$x = 4$$

9. Solve the following questions and represent their solutions on real line.

$$3x - 5 = 7$$

Solution:

$$3x = 7 + 5$$

$$3x = 12$$

$$x = \frac{12}{3} = 4$$

Check: Substitute $x = 4$ into the original equation

$$3(4) - 5 = 7$$

$$12 - 5 = 7$$

$$7 = 7$$

So, $x = 4$ is a solution because it makes the original equation true.

Representation of the solution on a number line:



10. Solve the following questions and represent their solutions on real line.

$$\frac{x-2}{5} - \frac{x-4}{2} = 2$$

$$\frac{2(x-2) - 5(x-4)}{10} = 2$$

$$\frac{2x-4-5x+20}{10} = 2$$

$$\frac{-3x+16}{10} = 2$$

$$-3x+16 = 2 \times 10$$

$$-3x+16 = 20$$

$$-3x = 20 - 16$$

$$-3x = 4$$

$$x = \frac{-4}{3}$$

Check: Substitute $x = \frac{-4}{3}$ into the original equation

$$\frac{\frac{-4}{3} - 2}{5} - \frac{\frac{-4}{3} - 4}{2} = 2$$

$$\Rightarrow \frac{-4-6}{5} - \frac{-4-12}{2} = 2$$

$$\Rightarrow \frac{-10}{5} - \frac{-16}{2} = 2$$

$$\Rightarrow -\frac{2}{1} + \frac{8}{1} = 2$$

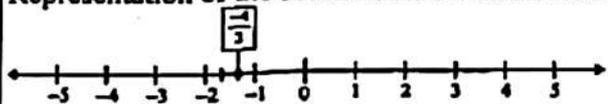
$$\Rightarrow \frac{-2+8}{1} = 2$$

$$\Rightarrow \frac{6}{1} = 2$$

$$\Rightarrow 6 = 2$$

So, $x = \frac{-4}{3}$ is the solution of given equation.

Representation of the solution on a number line:



11. Find solution of $\frac{2}{3}x - 1 < 0$ and also represent it on a real line.

Solution:

$$\frac{2}{3}x - 1 < 0$$

$$\Rightarrow \frac{2}{3}x < 1$$

$$\Rightarrow 2x < 3$$

$$\Rightarrow x < \frac{3}{2}$$

It means that all real numbers less than $\frac{3}{2}$ are in the solution

Thus the interval $(-\infty, \frac{3}{2})$ or $-\infty < x < \frac{3}{2}$ is the solution of the given inequality.

$$x < 7$$

It means all real numbers less than 7 are the solution of this inequality.



18. Solve inequality and represent the solution on a real line. $3 + 2x \geq 3$

Solution:

$$3 + 2x \geq 3$$

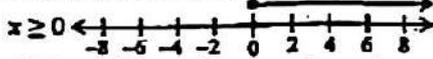
$$2x \geq 3 - 3$$

$$2x \geq 0$$

$$x \geq \frac{0}{2}$$

$$x \geq 0$$

It means all real numbers greater than or equal to 0 are the solution of this inequality.



19. Solve inequality and represent the solution on a real line. $6(x + 10) \leq 0$

Solution:

$$6(x + 10) \leq 0$$

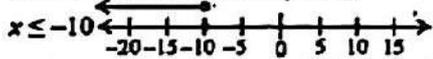
$$6x + 60 \leq 0$$

$$6x \leq -60$$

$$x \leq -\frac{60}{6}$$

$$x \leq -10$$

It means all real numbers less than or equal to -10 are the solution of this inequality.



20. Solve and represent their solutions on

real line. $\frac{x+5}{3} = 1-x$

Solution:

$$\frac{x+5}{3} = 1-x$$

$$1-3 \times \frac{(x+5)}{3} = 3(1-x)$$

$$x+5 = 3-3x$$

$$x+3x = 3-5$$

$$4x = -2$$

$$x = -\frac{2}{4}$$

$$x = -\frac{1}{2}$$

Check put $x = -\frac{1}{2}$ into the original equation

$$\frac{-\frac{1}{2}+5}{3} = 1 - \left(-\frac{1}{2}\right)$$

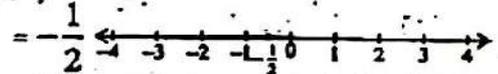
$$\frac{-1+10}{3} = 1 + \frac{1}{2}$$

$$\frac{9}{3} = \frac{2+1}{2}$$

$$\frac{9}{2} \times \frac{1}{3} = \frac{3}{2}$$

$$\frac{3}{2} = \frac{3}{2} \quad (\text{which is true})$$

So, $x = -\frac{1}{2}$ is solution.



21. Solve and represent their solutions on real line. $3x + 7 < 16$.

Solution:

$$3x + 7 < 16$$

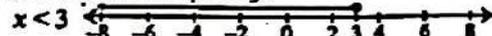
$$3x < 16 - 7$$

$$3x < 9$$

$$x < \frac{9}{3}$$

$$x < 3$$

It means all the real numbers less than 3 are the solution of this inequality.



22. Solve and represent their solutions on real line. $5(x-3) \geq 26x - (10x+4)$

Solution:

$$5(x-3) \geq 26x - (10x+4)$$

$$5x - 15 \geq 26x - 10x - 4$$

$$5x - 15 \geq 16x - 4$$

$$5x - 16x \geq -4 + 15$$

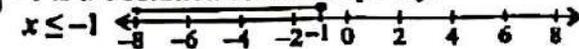
$$-11x \geq 11$$

$$11x \leq -11$$

$$x \leq \frac{-11}{11}$$

$$x \leq -1$$

It means all the real number less than or equal to -1 is the solution of this inequality.



Unit-5

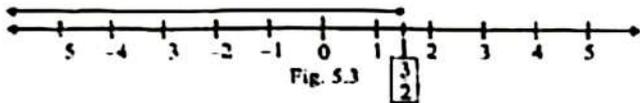
LINEAR EQUATIONS AND INEQUALITIES

Short Questions

Shade the solution region for linear inequalities in xy -plane: $2x + 1 \geq 0$

Indicate the solution region of linear inequalities by shading:

$$2x - 3y \leq 6, 2x + 3y \leq 12$$



We conclude that the solution of an inequality consists of all solutions of the inequality.

12. Solve and represent the solution on a real line. $12x + 30 = -6$

Solution:

$$12x + 30 = -6$$

$$12x = -6 - 30$$

$$12x = -36$$

$$x = \frac{-36}{12}$$

$$x = -3$$

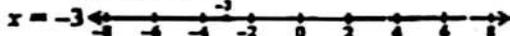
Check: Put $x = -3$ into the original equation

$$12(-3) + 30 = -6$$

$$-36 + 30 = -6$$

$$-6 = -6 \text{ (Which is true)}$$

So, $x = -3$ is solution



13. Solve and represent the solution on a real line. $\frac{x}{3} + 6 = -12$

Solution:

$$\frac{x}{3} + 6 = -12$$

$$\frac{x}{3} = -12 - 6$$

$$\frac{x}{3} = -18$$

$$\frac{x}{3} \times 3 = -18 \times 3$$

$$x = -54$$

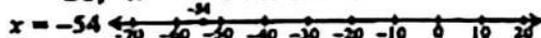
Check: Put $x = -54$ into the original equation

$$\frac{-54}{3} + 6 = -12$$

$$-18 + 6 = -12$$

$$-12 = -12 \text{ (Which is true)}$$

So, $x = -54$ is solution



14. Solve and represent the solution on a real line. $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$

Solution:

$$\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$$

$$\frac{2(x) - 3x(1)}{4} = \frac{1}{12}$$

$$\frac{2x - 3x}{4} = \frac{1}{12}$$

$$-x = 4 \times \frac{1}{12}$$

$$-x = \frac{1}{3}$$

$$x = -\frac{1}{3}$$

Check: Put $x = -\frac{1}{3}$ into the original equation

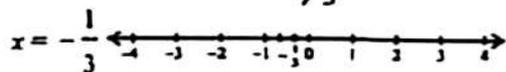
$$\frac{-\frac{1}{3}}{2} - \frac{3(-\frac{1}{3})}{4} = \frac{1}{12}$$

$$\frac{-1}{6} + \frac{1}{4} = \frac{1}{12}$$

$$\frac{-2+3}{12} = \frac{1}{12}$$

$$\frac{1}{12} = \frac{1}{12} \text{ (Which is true)}$$

So, $x = -\frac{1}{3}$ is solution



15. Solve and represent the solution on a real line. $2 = 7(2x + 4) + 12x$

Solution:

$$2 = 7(2x + 4) + 12x$$

$$2 = 14x + 28 + 12x$$

$$2 - 28 = 14x + 12x$$

$$-26 = 26x$$

$$\frac{-26}{26} = x$$

$$-1 = x$$

$$x = -1$$

Check put $x = -1$ into the original equation

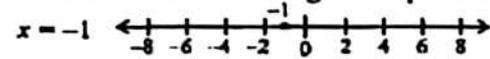
$$2 = 7[2(-1) + 4] + 12(-1)$$

$$2 = 7[-2 + 4] - 12$$

$$2 = 7(2) - 12$$

$$2 = 2 \text{ (Which is true)}$$

So, $x = -1$ is solution of the given equation.



16. Solve inequality and represent the solution on a real line. $x - 6 \leq -2$

Solution:

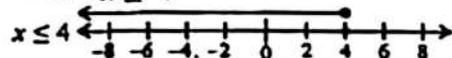
$$x - 6 \leq -2$$

$$x \leq -2 + 6$$

$$x \leq 4$$

It means all real numbers less than and equal to 4 are the solution of this inequality.

$$-\infty < x \leq 4$$



17. Solve inequality and represent the solution on a real line. $-9 > -16 + x$

Solution:

$$-9 > -16 + x$$

$$-9 + 16 > x$$

3. Find the solution region by drawing the graph the system of inequalities
 $x - 2y \leq 6$; $2x + y \geq 2$
4. Shade the solution region for linear inequalities in xy -plane: $5x - 4y \leq 20$
5. Shade the solution region for linear inequalities in xy -plane: $2x + 1 \geq 0$
6. Shade the solution region for linear inequalities in xy -plane: $3y - 4 \leq 0$
7. Indicate the solution region of linear inequalities by shading:
 $x + y \geq 5$, $-y + x \leq 1$
8. Indicate the solution region of linear inequalities by shading:
 $3x + 7y \geq 21$, $x - y \leq 2$
9. Indicate the solution region of linear inequalities by shading:
 $4x - 3y \leq 12$, $x \geq -\frac{3}{2}$
10. Indicate the solution region of linear inequalities by shading:
 $3x + 7y \geq 21$, $y \leq 4$
11. Indicate the solution region of linear inequalities by shading:
 $5x + 7y \leq 35$, $x - 2y \leq 2$

Unit-6

TRIGONOMETRY MCQs

1. The value of $\theta \tan^{-1} 2$ in radians is.
 (a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) 1.11π (d) 1.11
2. In a right triangle, the hypotenuse is 13 units and one of the angles is $\theta = 30^\circ$.
 What is the length of the opposite side?
 (a) 6.5 units (b) 7.5 units (c) 6 units (d) 5 units
3. In a right-angled triangle, if one angle is 45° , what is the length of the hypotenuse if
 one side is 5 cm?
 (a) $5\sqrt{2}$ cm (b) 5 cm (c) 10 cm (d) $10\sqrt{2}$
4. Which of the following is equivalent to $\sec^2 \theta - \tan^2 \theta$?
 (a) $\sin^2 \theta$ (b) 1 (c) $\cos^2 \theta$ (d) $\cot^2 \theta$
5. If $\sin \theta = \frac{3}{5}$, and θ is an acute angle, what is the value of $\cos^2 \theta$?
 (a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) $\frac{16}{25}$ (d) $\frac{4}{25}$
6. $\frac{5\pi}{24}$ rad = _____ degrees.
 (a) 30° (b) 37.5° (c) 45° (d) 52.5°
7. What is the radian measure of an angle 292.5° ?
 (a) $\frac{17\pi}{6}$ (b) $\frac{17\pi}{4}$ (c) 1.6π (d) 1.625π
8. Which of the following is a valid identity?
 (a) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$ (b) $\cos\left(\frac{\pi}{2} - \theta\right) = \cos \theta$
 (c) $\cos\left(\frac{\pi}{2} - \theta\right) = \sec \theta$ (d) $\cos\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec} \theta$

9. $\sin 60^\circ =$ _____
 (a) 1 (b) $\frac{1}{2}$ (c) $\sqrt{(3)^2}$ (d) $\frac{\sqrt{3}}{2}$
10. $\cos^2 100\pi + \sin^2 100\pi =$ _____
 (a) 1 (b) 2 (c) 3 (d) 4
11. In $\angle AOB$ vertex is.
 (a) A (b) O (c) B (d) none of these
12. Co-terminal of 65°
 (a) 360° (b) -360° (c) 425° (d) -405°
13. In right angled triangle ABC if $m\angle B = 90^\circ$ then which statement is true.
 (a) $a^2 + b^2 = c^2$ (b) $a^2 - b^2 = c^2$ (c) $a^2 + c^2 = b^2$ (d) $b^2 + c^2 = a^2$
14. One full revolution of a circle is _____ radian.
 (a) π (b) 2π (c) 3π (d) 4π
15. Area of sector is = _____
 (a) $4\pi r^2$ (b) πr^2 (c) $\frac{1}{2}r^2\theta$ (d) $\frac{1}{2}r^2\theta$
16. Reciprocal of $\sin \theta$
 (a) $\sec \theta$ (b) $\operatorname{cosec} \theta$ (c) $\cot \theta$ (d) $\tan \theta$
17. $\sin \theta =$ _____
 (a) $\sqrt{1 - \cos^2 \theta}$ (b) $\frac{1}{\tan \theta}$ (c) $\frac{\sin \theta}{\cos \theta}$ (d) $1 - \cos^2 \theta$
18. If $\sin \theta = \frac{2}{3}$ then value of $\operatorname{cosec} \theta$
 (a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$
19. What is the measure of a right angle in radians?
 (a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π
20. Convert 15° to radians.
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{12}$
21. Which of the following is a Pythagorean identity?
 (a) $\sin^2 \theta + \cos^2 \theta = 1$ (b) $\sin \theta + \cos \theta = 1$ (c) $\tan \theta + \cot \theta = 1$ (d) $\sec \theta + \operatorname{cosec} \theta = 1$
22. What is the value of $\sin 45^\circ$?
 (a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$
23. What is the value of $\cos 30^\circ$?
 (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1
24. In a right-angled triangle, if one angle is 30° , what is the other acute angle?
 (a) 60° (b) 45° (c) 90° (d) 120°

TRIGONOMETRY

Short Questions

Exercise 6.1

1. What is trigonometry?

Ans: Trigonometry: Trigonometry is a branch of mathematics that deals with the relationships between the angles and sides of triangles, especially right-angled triangles.

2. Define Angle.

Ans: Angle: A plane figure which is formed by two rays sharing a common end point is called angle. The two rays (\overrightarrow{OA} and \overrightarrow{OB}) are known as the sides of the angle. The common end point "O" is known as vertex.

3. Define acute angle, and write it in symbolically.

Ans: Acute angle: An angle which is less than 90° is called an acute angle. Acute angle in symbolically $0 < \theta < 90^\circ$.

4. Define obtuse angle also write it in symbolically.

Ans: Obtuse angle: An angle which is greater than 90° and less than 180° is called an obtuse angle. Obtuse angle in symbolically $90^\circ < \theta < 180^\circ$.

5. Define right angle.

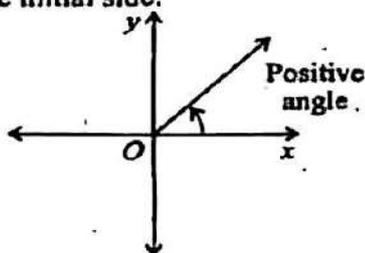
Ans: Right angle: An angle which is equal to 90° is called a right angle.

6. How many types of an angle name them.

Ans: There are six types of angles.
i. Acute angle ii. Obtuse angle
iii. Right angle iv. Straight angle
v. Reflex angle vi. Full rotation

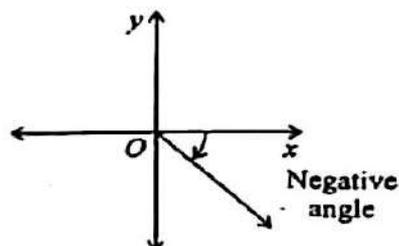
7. Define positive angle.

Ans: Positive angle: The angle will be positive if the terminal side is rotated counterclockwise (anti clockwise) from the initial side.



8. Define negative angle.

Ans: Negative angle: The angle will be negative if the terminal side is rotated clockwise from the initial side.



9. Define degree.

Ans: Degree: A degree ($^\circ$) is a unit of measurement of angles. It represents one $\frac{1}{360}$ of a full rotation around a point. In simpler terms, a degree is the measure of an angle, with a complete circle being 360° .

10. Define radian.

Ans: Radian: A radian is a unit of angular measure in mathematics, particularly in trigonometry. It is defined as, "the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle".

11. What is the relationship between degree and radian

Ans: Radians to Degrees:

$$1 \text{ rad} = \frac{180^\circ}{\pi} \text{ degrees, } \approx 57.295^\circ$$

Degrees to Radians:

$$1^\circ = \frac{\pi}{180^\circ} \text{ rad } \approx 0.0175 \text{ radian}$$

12. Convert radians to degree: $\frac{5\pi}{3}$ rad

Solution:

$$\frac{5\pi}{3} \text{ rad} = \frac{5\pi}{3} \times \frac{180^\circ}{\pi} = 300^\circ$$

13. Convert degree to radian: $15^\circ 15'$

Ans:

$$15^\circ 15' = 15^\circ + \frac{15}{60} = 15.25^\circ$$

$$= 15.25 \times \frac{\pi}{180} \text{ or } 0.266 \text{ rad}$$

14. Write formula area of sector .

Ans:

$$\text{Area of Sector} = \frac{1}{2} r^2 \theta$$

15. Find area of sector with $r = 8\text{cm}$ and $\theta = 45^\circ$.

Solution:

$$r = 8\text{cm}, \theta = 45^\circ$$

$$= 45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4} \text{ radians.}$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = 8\pi\text{cm}^2 \text{ or } 25.12\text{cm}^2.$$

The area of the sector is approximately 25.12cm^2 .

16. Find the arc length of a sector with radius $r = 10\text{cm}$ and central angle $\theta = 60^\circ$.

Solution:

$$r = 10, \theta = 60^\circ$$

$$= 60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3} \text{ radians.}$$

$$l = r\theta = 10 \times \frac{\pi}{3} = 10.47\text{cm}$$

The arc length is approximately 10.47cm

17. Find in which quadrant the following angles lie. Write a co-terminal angle for each:

(i) 65° (ii) 135°

Solution:

(i) 65°
 65° lie in first quadrant
 Co-terminal angle = 425°

(ii) 135°
 135° lie in second quadrant
 Co-terminal angle = -225°

18. Convert the following into to degrees, minutes, and seconds. 123.456°

Solution:

$$123.456^\circ$$

$$= 123^\circ.456'$$

$$= 123^\circ + (0.456 \times 60)''$$

$$= 123^\circ + 27.36'' = 123^\circ + 27' + 0.36''$$

$$= 123^\circ + 27' + (0.36 \times 60)''$$

$$= 123^\circ + 27' + 21.6''$$

$$= 123^\circ + 27' + 22''$$

$$= 123^\circ 27' 22''$$

19. Convert the following into decimal degrees. $65^\circ 32' 15''$

Solution:

$$65^\circ 32' 15''$$

$$= 65^\circ 32' 15'' = 65^\circ + 32' + 15''$$

$$= 65^\circ + \left(\frac{32}{60}\right)^\circ + \left(\frac{15}{60 \times 60}\right)^\circ$$

$$= 65^\circ + 0.53^\circ + 0.0041^\circ$$

$$= 65.5341^\circ$$

20. Convert into radian: 36°

Solution:

$$36^\circ$$

$$= 36 \times \frac{\pi}{180} = \frac{\pi}{5} \text{ radian}$$

21. Convert into degrees. $\frac{\pi}{16} \text{ rad}$

Solution:

$$\frac{\pi}{16} \text{ rad}$$

$$= \frac{\pi}{16} \times \frac{180}{\pi} = \frac{180}{16}$$

$$= 11.25^\circ$$

22. Find the arc length and area of a sector with:

$r = 6\text{cm}$ and central angle $\theta = \frac{\pi}{3}$ radians.

Solution:

Data:

$$r = 6\text{cm}$$

$$\theta = \frac{\pi}{3} \text{ rad}$$

$$l = ?$$

$$\text{Area of Sector} = ?$$

We know that:

$$l = r\theta$$

$$l = 6 \times \frac{\pi}{3}$$

$$l = 2\pi \text{ or } 6.28\text{cm}$$

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2}(6)^2 \times \frac{\pi}{3}$$

$$= \frac{1}{2}6 \times 6 \times \frac{\pi}{3}$$

$$= 6\pi\text{cm}^2$$

$$= 18.85\text{cm}^2$$

Exercise 6.2

23. Write reciprocal identities

Ans:

(i) $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ (ii) $\sec \theta = \frac{1}{\cos \theta}$

(iii) $\cot \theta = \frac{1}{\tan \theta}$

24. State fundamental trigonometric identities

Ans: Fundamental Trigonometric Identities

- (i) $\sin^2 \theta + \cos^2 \theta = 1$
- (ii) $\tan^2 \theta + 1 = \sec^2 \theta$
- (iii) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

25. Define Pythagoras theorem

Ans: In a right-angle triangle length of square of hypotenuse is equal sum of squares length of other two sides.

Exercise 6.3

26. Prove that $(\sec^2 \theta - 1) \cos^2 \theta = \sin^2 \theta$

Solution:

$$\begin{aligned} \text{L.H.S} &= (\sec^2 \theta - 1) \cos^2 \theta \\ &= \tan^2 \theta \cdot \cos^2 \theta \quad (\because 1 + \tan^2 \theta = \sec^2 \theta) \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \cos^2 \theta \quad \left(\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right) \\ &= \sin^2 \theta = \text{R.H.S} \end{aligned}$$

Hence, $(\sec^2 \theta - 1) \cos^2 \theta = \sin^2 \theta$

27. Show that $\tan \theta + \cot \theta = \sec \theta \operatorname{cosec} \theta$

Solution:

$$\begin{aligned} \text{L.H.S} &= \tan \theta + \cot \theta \\ &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin \theta \cdot \sin \theta + \cos \theta \cdot \cos \theta}{\cos \theta \cdot \sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} \quad (\because \sin^2 \theta + \cos^2 \theta = 1) \\ &= \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} \\ &= \sec \theta \cdot \operatorname{cosec} \theta = \text{R.H.S.} \end{aligned}$$

Hence, $\tan \theta + \cot \theta = \sec \theta \operatorname{cosec} \theta$

28. Prove that $\cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$

Solution:

$$\begin{aligned} \text{L.H.S} &= \cos^2 \theta - \sin^2 \theta \\ &= \cos^2 \theta - (1 - \cos^2 \theta) \\ &= \cos^2 \theta - 1 + \cos^2 \theta \\ &= 2\cos^2 \theta - 1 \end{aligned}$$

R.H.S Proved

29. Prove that $\cos^2 \theta - \sin^2 \theta = 1 - 2\sin^2 \theta$

Solution:

$$\begin{aligned} \text{L.H.S} &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - \sin^2 \theta - \sin^2 \theta \\ &= 1 - 2\sin^2 \theta \end{aligned}$$

R.H.S Proved

30. Prove that: $\sin \theta (\operatorname{cosec} \theta - \sin \theta) = \frac{1}{\sec^2 \theta}$

Solution:

$$\begin{aligned} \text{L.H.S} &= \sin \theta (\operatorname{cosec} \theta - \sin \theta) \\ &= \sin \theta \left(\frac{1}{\sin \theta} - \sin \theta \right) \\ &= \sin \theta \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right) \\ &= 1 - \sin^2 \theta \\ &= \cos^2 \theta \\ &= \frac{1}{\sec^2 \theta} \end{aligned}$$

31. Prove that:

$$(\sin \theta + \cos \theta)^2 = 1 + 2 \sin \theta \cos \theta$$

Solution:

$$(\sin \theta + \cos \theta)^2 = 1 + 2 \sin \theta \cos \theta$$

$$\begin{aligned} \text{L.H.S} &= (\sin \theta + \cos \theta)^2 \\ &= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta \\ &= 1 + 2 \sin \theta \cos \theta \end{aligned}$$

R.H.S Proved

32. Prove that: $\frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$

Solution:

$$\begin{aligned} \text{L.H.S} &= \frac{\cos \theta}{\sin \theta} \\ &= \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\tan \theta} \\ &= \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta} \\ &= \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= \frac{1}{\tan^2 \theta} \end{aligned}$$

R.H.S Proved

Exercise 6.4

33. Find the value of: $2 \sin 60^\circ \cos 60^\circ$

Solution:

$$\begin{aligned} &2 \sin 60^\circ \cos 60^\circ \\ &= 2 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2} \right) \end{aligned}$$

$$= \frac{\sqrt{3}}{2}$$

34. Find the value of:

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

Solution:

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$\begin{aligned}
 &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\
 &= \frac{\sqrt{3^2}}{4} + \frac{1}{4} \\
 &= \frac{3}{4} + \frac{1}{4} \\
 &= \frac{4}{4} \\
 &= 1
 \end{aligned}$$

Solution:

$$\begin{aligned}
 &3 \cos 45^\circ + 4 \sin 45^\circ \\
 &= 3\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{\sqrt{2}}\right) \\
 &= \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{2}} \\
 &= \frac{3+4}{\sqrt{2}} \\
 &= \frac{7}{\sqrt{2}}
 \end{aligned}$$

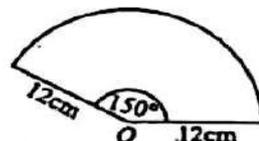
35. If $\sin \frac{\pi}{4}$ and $\cos \frac{\pi}{4}$ equal to $\frac{1}{\sqrt{2}}$, then find the value of: $3 \cos 45^\circ + 4 \sin 45^\circ$

Unit-6

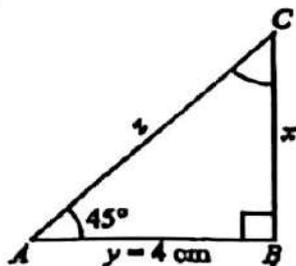
TRIGONOMETRY

Long Questions

1. A circular sector of radius $r = 12$ has an angle of 150° . This sector is cut out and then bent to form a cone. What is the slant height and the radius of the base of this cone?



2. If $\tan \theta = \frac{3}{4}$, find the remaining trigonometric ratios, when θ lies in first quadrant.
3. If $\sin \theta = \frac{2}{3}$ find the remaining trigonometric ratios, when θ lies in first quadrant.
4. If $\tan \theta = \frac{1}{2}$ find the remaining trigonometric ratios, when θ lies in first quadrant.
5. If $\cot \theta = \sqrt{\frac{3}{2}}$ find the remaining trigonometric ratios, when θ lies in first quadrant.
6. Prove that: $\frac{1 - \sin \theta}{\cos \theta} = \frac{\cos \theta}{1 + \sin \theta}$
7. Prove that: $(\sec \theta - \tan \theta)^2 = \frac{1 - \sin \theta}{1 + \sin \theta}$
8. Prove that: $(\tan \theta + \cot \theta)^2 = \sec^2 \theta \operatorname{cosec}^2 \theta$
9. Prove the trigonometric identities: $\frac{\sin \theta}{1 - \cos \theta} = \frac{1 + \cos \theta}{\sin \theta}$
10. Prove that: $\frac{\sin \theta}{1 - \cos \theta} = \frac{1 + \cos \theta}{\sin \theta}$
11. Show that $\tan \theta + \cot \theta = \sec \theta \operatorname{cosec} \theta$
12. Find the values of x , y and z :



13. Solve the triangles when $m\angle B = 90^\circ$: $m\angle C = 60^\circ$, $c = 3\sqrt{3}$ cm

14. Solve the triangles when $m\angle B = 90^\circ$: $a = 12\text{ cm}$, $c = 6\text{ cm}$
 15. Solve the triangles when $m\angle B = 90^\circ$: $m\angle A = 30^\circ$, $c = 4\text{ cm}$
 16. Solve triangle ABC , when $a = \sqrt{2}\text{ cm}$, $c = 1\text{ cm}$ and $m\angle B = 90^\circ$
 17. Solve triangle ABC , when $m\angle A = 60^\circ$, $b = 5\text{ cm}$, $m\angle B = 90^\circ$

Unit-7

COORDINATE GEOMETRY MCQs

1. The equation of a straight line in the slope-intercept form is written as:
 (a) $y = m(x + c)$ (b) $y - y_1 = m(x - x_1)$ (c) $y = c + mx$ (d) $ax + by + c = 0$
2. The gradients of two parallel lines are:
 (a) equal (b) zero
 (c) negative reciprocals of each other (d) always undefined
3. If the product of the gradients of two lines is -1 , then the lines are:
 (a) parallel (b) perpendicular (c) collinear (d) coincident
4. Distance between two points $P(1, 2)$ and $Q(4, 6)$ is:
 (a) 5 (b) 6 (c) $\sqrt{13}$ (d) 4
5. The midpoint of a line segment with endpoints $(-2, 4)$ and $(6, -2)$ is:
 (a) $(4, 2)$ (b) $(2, 1)$ (c) $(1, 1)$ (d) $(0, 0)$
6. A line passing through points $(1, 2)$ and $(4, 5)$ is:
 (a) $y = x + 1$ (b) $y = 2x + 3$ (c) $y = 3x - 2$ (d) $y = x + 2$
7. The equation of line in point-slope form is:
 (a) $y = m(x + c)$ (b) $y - y_1 = m(x - x_1)$ (c) $y = c + mx$ (d) $ax + by + c = 0$
8. $2x + 3y - 6 = 0$ in the slope-intercept form is:
 (a) $y = \frac{-2}{3}x + 2$ (b) $y = \frac{2}{3}x - 2$ (c) $y = \frac{2}{3}x + 1$ (d) $y = \frac{-2}{3}x - 2$
9. The equation of a line in symmetric form is:
 (a) $\frac{x}{a} + \frac{y}{b} = 1$ (b) $\frac{x - x_1}{1} + \frac{y - y_1}{m} = \frac{z - z_1}{1}$
 (c) $ax + by + c = 0$ (d) $y - y_1 = m(x - x_1)$
10. The equation of a line in normal form is:
 (a) $y = mx + c$ (b) $\frac{x}{a} + \frac{y}{b} = 1$ (c) $\frac{x - x_1}{\cos\alpha} - \frac{y - y_1}{\sin\alpha}$ (d) $x\cos\alpha + y\sin\alpha = p$
11. In a plane horizontal line represent.
 (a) xox' (b) yoy' (c) zoz' (d) none of these
12. In a plane yoy' is called _____:
 (a) horizontal line (b) vertical line (c) parallel line (d) none of these
13. In (x, y) if $x > 0$ and $y < 0$ then point in which quadrant line.
 (a) I (b) II (c) III (d) IV
14. The point $P(3, -4)$ lies in which quadrant?
 (a) I (b) II (c) III (d) IV
15. The distance between the points $A(2, 3)$ and $B(5, 7)$ is:
 (a) 5 (b) 6 (c) 7 (d) 8
16. The midpoint of the line segment joining $(1, 2)$ and $(5, 8)$ is:
 (a) $(3, 5)$ (b) $(4, 5)$ (c) $(3, 4)$ (d) $(4, 6)$
17. The point $(0, 5)$ lies on which axis?
 (a) x-axis (b) y-axis (c) Both axes (d) None
18. If the slope of a line is undefined, the line is:
 (a) Horizontal (b) Vertical (c) Parallel to x-axis (d) Parallel to y-axis
19. The distance between two towns with coordinates $(3, 4)$ and $(7, 1)$ is:
 (a) 6 (b) 5 (c) 7 (d) 8

20. The midpoint of a bridge connecting (2, 5) and (8, 9) is:
 (a) (5, 7) (b) (6, 7) (c) (5, 8) (d) (6, 8)

Unit-7

COORDINATE GEOMETRY

Short Questions

Exercise 7.1

1. Define coordinate plane.

Ans: **Coordinate plane:** A coordinate plane is a two-dimensional surface formed by two perpendicular number lines:
 The x-axis (horizontal line)
 The y-axis (vertical line)
 These axes intersect at the origin $O(0,0)$ dividing the plane into four quadrants. The coordinate plane is used to plot points, graph equations, and represent geometric shapes using ordered pairs (x, y) .

2. Define origin.

Ans: **Origin:** Point of intersection of both axis is called origin. It is denoted by $O(0,0)$.

3. Define Abscissa and ordinate.

Ans: **Abscissa:** The first component of the ordered pair (x, y) is called x-coordinate or abscissa.

Ordinate: ordinate the second component is called y-coordinate or ordinate.

4. Define co-ordinate axis.

Ans: **Co-ordinate axis:** The two lines $x'Ox$ and $y'Oy$ are called the coordinate axes. The horizontal line $x'Ox$ is called the x-axis and the vertical line $y'Oy$ is called the y-axis.

5. How many quadrant divide in a co-ordinate plane.

Ans: The coordinate plane is divided into four quadrants based on the signs of x and y:

Quadrant I: $(+x, +y)$ – Both coordinates are positive.

Quadrant II: $(-x, +y)$ – x is negative, y is positive.

Quadrant III: $(-x, -y)$ Both coordinates are negative.

Quadrant IV: $(+x, -y)$ x is positive, -y is negative.

6. What is distance formula.

Ans: The distance between two points

$A(x_1, y_1)$ and $B(x_2, y_2)$ in a coordinate plane is given by the formula:

$$d = |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

7. What is midpoint formula.

Ans: The midpoint of a line segment joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is calculated using the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

8. Find the distance between two points $A(3, 4)$ and $B(7, 1)$.

Solution:

Using the distance formula:

$$|AB| = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting the given values:

$$d = \sqrt{(7-3)^2 + (1-4)^2}$$

$$d = \sqrt{(4)^2 + (-3)^2}$$

$$d = \sqrt{16+9} = \sqrt{25} = 5 \text{ units.}$$

So, the distance between A and B is 5 units.

9. Find the midpoint of the line segment joining $A(2, 6)$ and $B(8, 10)$.

Solution:

Using the midpoint formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Substituting the values:

$$M = \left(\frac{2+8}{2}, \frac{6+10}{2} \right) = \left(\frac{10}{2}, \frac{16}{2} \right)$$

$$M = (5, 8)$$

So, the midpoint is (5, 8).

10. Show that the points $A(-1, 2)$, $B(7, 5)$ and $C(2, -6)$ are vertices of a right triangle.

Solution:

Let a, b and c denote the lengths of the sides BC, CA and AB respectively.

By using the distance formula, we have

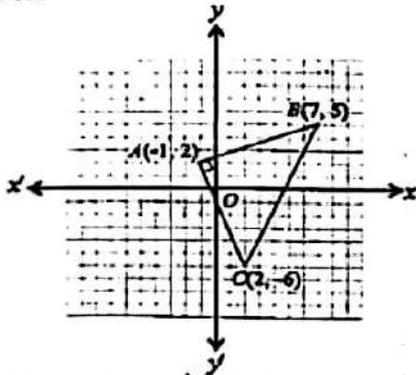
$$c = |\overline{AB}| = \sqrt{(7 - (-1))^2 + (5 - 2)^2} = \sqrt{73}$$

$$a = |\overline{BC}| = \sqrt{(2 - 7)^2 + (-6 - 5)^2} = \sqrt{146}$$

$$b = |\overline{CA}| = \sqrt{(2 - (-1))^2 + (-6 - 2)^2} = \sqrt{73}$$

Clearly: $a^2 = b^2 + c^2$

Therefore, ABC is a right triangle with right angle at A .

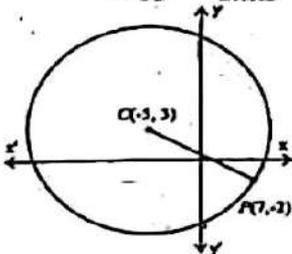


11. The point $C(-5, 3)$ is the centre of a circle and $P(7, -2)$ lies on the circle. What is the radius of the circle?

Solution:

The radius of the circle is the distance from C to P . By using distance formula, we have

$$\begin{aligned} \text{Radius} = |\overline{CP}| &= \sqrt{[7 - (-5)]^2 + (-2 - 3)^2} \\ &= \sqrt{144 + 25} = \sqrt{169} \\ &= 13 \text{ units} \end{aligned}$$



12. Describe the location in the plane of the point $P(x, y)$, for which $x > 0$ and $y > 0$.

Solution:

$$x > 0 \text{ and } y > 0$$

$x > 0$ and $y > 0$ lies in 1st quadrant.

13. Find the distance between the points: $P(-8, -7)$, $Q(0, 0)$.

Solution:

$$P(-8, -7), Q(0, 0)$$

$$|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(0 + 8)^2 + (0 + 7)^2}$$

$$d = \sqrt{(8)^2 + (7)^2}$$

$$d = \sqrt{64 + 49}$$

$$d = \sqrt{113} \text{ units}$$

14. Find the distance between the two given points.

$$A(-8, 3), B(2, -1)$$

Solution:

Distance between the two given points

$$A(-8, 3), B(2, -1)$$

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 + 8)^2 + (-1 - 3)^2}$$

$$d = \sqrt{(10)^2 + (-4)^2}$$

$$d = \sqrt{100 + 16}$$

$$d = \sqrt{116}$$

$$d = \sqrt{2^2 \times 29}$$

$$d = 2\sqrt{29} \text{ units}$$

15. Find the distance between the points: $A(6, 7)$, $B(0, -2)$

Solution:

$$A(6, 7), B(0, -2)$$

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(0 - 6)^2 + (-2 - 7)^2}$$

$$d = \sqrt{(-6)^2 + (-9)^2}$$

$$d = \sqrt{36 + 81}$$

$$d = \sqrt{117}$$

$$d = \sqrt{3^2 \times 13}$$

$$d = 3\sqrt{13} \text{ units}$$

16. Find the midpoint of the line segment joining the two points:

$$A(-8, 3), B(2, -1)$$

Solution:

$$A(-8, 3), B(2, -1)$$

$$\text{Mid-point of } \overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-8 + 2}{2}, \frac{3 - 1}{2} \right)$$

$$= \left(\frac{-6}{2}, \frac{2}{2} \right)$$

$$= (-3, 1)$$

17. Find the midpoint of the line segment joining the two points:

$$A(3, 1), B(-2, -4)$$

Solution:

$$\text{Mid-point of } \overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{3 - 2}{2}, \frac{1 - 4}{2} \right)$$

$$= \left(\frac{1}{2}, \frac{-3}{2} \right)$$

18. Find h such that $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.

Solution:

Since the points A, B, C are collinear, So
Slope of AB = slope of BC

$$\frac{h-2}{-1-3} = \frac{2-3}{3-7} \quad \left(\because \frac{y_2 - y_1}{x_2 - x_1} = m \right)$$

$$\frac{h-2}{-4} = \frac{-1}{-4}$$

$$h-2 = -1$$

$$h = -1 + 2$$

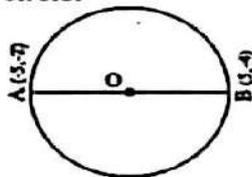
$$\boxed{h=1}$$

19. The points $A(-5, -2)$ and $B(5, -4)$ are ends of a diameter of a circle. Find the centre and radius of the circle.

Solution:

Let O be the centre of the circle and it is the midpoint of the end points of a diameter.

Let consider a circle.



$$\begin{aligned} M(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-5 + 5}{2}, \frac{-2 + (-4)}{2} \right) \\ &= \left(\frac{0}{2}, \frac{-6}{2} \right) = (0, -3) \end{aligned}$$

$m(x, y)$ = centre $O(0, -3)$

Radius of circle is the distance \overline{OA} or \overline{OB}

Radius = $|\overline{OB}|$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5-0)^2 + [-4 - (-3)]^2}$$

$$= \sqrt{(5)^2 + (-4+3)^2}$$

$$= \sqrt{25 + (-1)^2}$$

$$= \sqrt{25+1} = \sqrt{26}$$

Radius = $\sqrt{26}$

Exercise 7.2

20. Define collinear points.

Ans: Three or more points are collinear if they lie on the same straight line.

21. Define parallel lines.

Ans: Two lines are said to be parallel if they never intersect and have the same slope, i.e.,

$$m_1 = m_2$$

22. Define perpendicular lines.

Ans: Two lines are perpendicular if they intersect at a 90° angle. Their slopes satisfy the condition: $m_1 \times m_2 = -1$

23. What is intercept form of a line?

Ans: A line that cuts the x -axis at $(a, 0)$ and the y -axis at $(0, b)$ follows the equation:

$$\frac{x}{a} + \frac{y}{b} = 1$$

24. Show that the points $A(-3, 6)$, $B(3, 2)$ and $C(6, 0)$ are collinear.

Solution:

We know that the points A, B and C are collinear if the line AB and BC have the same slopes.

$$\text{Here Slope of } AB = \frac{2-6}{3-(-3)} = \frac{-4}{3+3} = \frac{-4}{6} = \frac{-2}{3}$$

$$\text{and slope of } BC = \frac{0-2}{6-3} = \frac{-2}{3}$$

\therefore Slope of AB = Slope of BC

Thus A, B and C are collinear.

25. Write down an equation of the straight line passing through $(5, 1)$ and parallel to a line passing through the points $(0, -1)$, $(7, -15)$.

Solution:

Let m be the slope of the required straight line, then

$$m = \frac{-15 - (-1)}{7 - 0}$$

(\because Slopes of parallel lines are equal)

$$= -2$$

As the point $(5, 1)$ lies on the required line having slope -2 so, by point-slope form of equation of the straight line, we have

$$y - (1) = -2(x - 5)$$

$$\text{or } y = -2x + 11$$

$$\text{or } 2x + y - 11 = 0$$

is an equation of the required line.

26. Find an equation of line through the points $(-2, 1)$ and $(6, -4)$.

Solution:

Using two-points form of the equation of straight line, the required equation is

$$y - 1 = \frac{-4 - 1}{6 - (-2)} [x - (-2)]$$

$$\text{or } y - 1 = \frac{-5}{8} (x + 2) \text{ or } 5x + 8y + 2 = 0$$

27. Find the slope and inclination of the line joining the points: (3, -2); (2, 7)

Solution:

$$(3, -2); (2, 7)$$

$$\left(\begin{matrix} x_1 & y_1 \\ 3 & -2 \end{matrix} \right), \left(\begin{matrix} x_2 & y_2 \\ 2 & 7 \end{matrix} \right)$$

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{-2 - 7}{3 - 2} = \frac{-9}{1}$$

$$\boxed{\text{Slope} = -9}$$

$$\tan \theta = \text{slope}$$

$$\tan \theta = -9$$

$$\theta = \tan^{-1}(-9)$$

$$\boxed{\theta = -83.65^\circ}$$

So, $\theta = 180^\circ - 83.65$

$$\boxed{\theta = 96.34^\circ}$$

28. Find the slope and inclination of the line joining the points: (-2, 4); (5, 11)

Solution:

(i) (-2, 4); (5, 11)

$$\left(\begin{matrix} x_1 & y_1 \\ -2 & 4 \end{matrix} \right), \left(\begin{matrix} x_2 & y_2 \\ 5 & 11 \end{matrix} \right)$$

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{4 - 11}{-2 - 5}$$

$$= \frac{-7}{-7}$$

$$\boxed{\text{Slope} = 1}$$

$$\text{Slope} = \tan \theta$$

$$\tan \theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\boxed{\theta = 45^\circ}$$

29. Find the slope and inclination of the line joining the points: (4, 6); (4, 8)

Solution:

$$(4, 6); (4, 8)$$

$$\left(\begin{matrix} x_1 & y_1 \\ 4 & 6 \end{matrix} \right), \left(\begin{matrix} x_2 & y_2 \\ 4 & 8 \end{matrix} \right)$$

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{6 - 8}{4 - 4}$$

$$= \frac{-2}{0}$$

$$\boxed{\text{Slope} = \text{undefined}}$$

$$\tan \theta = \text{slope}$$

$$\tan \theta = \text{undefined}$$

$$\theta = \tan^{-1}(\infty)$$

$$\boxed{\theta = 90^\circ}$$

30. Two pairs of points are given. Find whether the two lines determined by these points are:

- (i) parallel (ii) perpendicular
(a) (1, -2), (2, 4) and (4, 1), (-8, 2)

Solution:

$$(1, -2), (2, 4) \text{ and } (4, 1), (-8, 2)$$

$$\text{Slope} = \frac{-2 - 4}{1 - 2} \text{ and } \text{Slope} = \frac{1 - 2}{4 - 8}$$

$$m_1 = \frac{-6}{-1} \text{ and } m_2 = \frac{-1}{12}$$

$$m_1 = 6 \text{ and } m_2 = -\frac{1}{12}$$

Now,

$$m_1 \neq m_2$$

$$m_1 m_2 \neq -1$$

So, m_1 & m_2 are neither parallel nor perpendicular.

31. Find the equation of line through (-4, -6) and perpendicular to line having slope $\frac{-3}{2}$

Solution:

$$\text{Point } (-4, -6)$$

$$x_1 = -4, \quad y = -6$$

$$\text{Slope} = \frac{-3}{2}$$

$$\text{Slope of } \perp \text{ Line} = \frac{-1}{\text{Slope}} \quad (m_1 m_2 = -1)$$

$$m = \frac{-1}{-3/2}$$

$$m = \frac{2}{3}$$

Now,

$$y - y_1 = m(x - x_1)$$

$$y + 6 = \frac{2}{3}(x + 4)$$

$$3(y + 6) = 2(x + 4)$$

$$3y + 18 = 2x + 8$$

$$2x - 3y = 18 - 8$$

$$2x - 3y = 10$$

32. Find an equation of the line through (11, -5) and parallel to a line with slope -24.

Solution:

$$x_1 = 11, \quad y_1 = -5$$

$$\text{Given Slope} = -24$$

$$\text{Slop of Parallel line} = m = -24$$

$$\begin{aligned}
 y - y_1 &= m(x - x_1) \\
 y + 5 &= -24(x - 11) \\
 y + 5 &= -24x + 264 \\
 24x + y &= 264 - 5 \\
 24x + y &= 259
 \end{aligned}$$

33. Write down an equation of the line which cuts the x -axis at $(2, 0)$ and y -axis at $(0, -4)$.

Solution:

As 2 and -4 are respectively x and y -intercepts of the required line, so by two-intercepts form of equation of a straight line, we have

$$\frac{x}{2} + \frac{y}{-4} = 1 \quad \text{or} \quad 2x - y - 4 = 0$$

Which is the required equation.

Unit-7

COORDINATE GEOMETRY

Long Questions

- Show that: the points $A(0, 2)$, $B(\sqrt{3}, 1)$ and $C(0, -2)$ are vertices of a right triangle.
- Find h such that $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.
- Show that points $A(3, 1)$, $B(-2, -3)$ and $C(2, 2)$ are vertices of an isosceles triangle.
- Show that points $A(5, 2)$, $B(-2, 3)$, $C(-3, -4)$ and $D(4, -5)$ are vertices of parallelogram.
- Find k so that the line joining $A(7, 3)$; $B(k, -6)$ and the line joining $C(-4, 5)$ $D(-6, 4)$ are: (i) parallel (ii) perpendicular.
- The points $A(-5, -2)$ and $B(5, -4)$ are ends of a diameter of a circle. Find the centre and radius of the circle.
- The length of perpendicular from the origin to a line is 5 units and the inclination of this perpendicular is 120° . Find the slope and y -intercept of the line.
- Transform the equation $5x - 12y + 39 = 0$ into Slope intercept form and Two-intercept form
- Using slopes, show that the triangle with its vertices $A(6, 1)$ $B(2, 7)$ and $C(-6, -7)$ is a right triangle.
- Find an equation of the perpendicular bisector of the segment joining the points $A(3, 5)$ and $B(9, 8)$.
- Convert the equations into slope intercept form, two intercept form and normal form. $2x - 4y + 11 = 0$
- Convert the equations into slope intercept form, two intercept form and normal form. $4x + 7y - 2 = 0$

Unit-9

SIMILAR FIGURES

MCQs

- Q.1. Four option are given against each statement. Encircle the correct option.
- If two polygons are similar, then:
 - their corresponding angles are equal.
 - their volumes are equal.
 - their corresponding angles are equal.
 - their corresponding sides are equal.
 - The ratio of the areas of two similar polygons is;
 - equal to the ratio of their perimeters.
 - equal to the square of the ratio of their corresponding sides.
 - equal to the cube of the ratio of their corresponding sides.
 - equal to the sum of their corresponding sides.
 - If the volume of two similar solids is 125 cm^3 and 27 cm^3 , the ratio of their corresponding heights is _____.
 - 3 : 5
 - 5 : 3
 - 25 : 9
 - 9 : 25
 - If base = 8cm and altitude = 4cm then area of parallelogram.
 - 12 cm^2
 - 4 cm^2
 - 18 cm^2
 - 32 cm^2

5. A parallelogram has an area of 64 cm^2 and a similar parallelogram has an area of 144 cm^2 . If a side of the smaller parallelogram is 8 cm , the corresponding side of the larger parallelogram is:
 (a) 10 cm (b) 12 cm (c) 18 cm (d) 16 cm
6. Area of parallelogram.
 (a) base + altitude (b) base \times altitude (c) $\frac{1}{2}$ (base \times altitude) (d) length \times width
7. Three are more than sided closed figure is called:
 (a) circle (b) rectangle (c) triangle (d) polygon
8. _____ figures have same shape but different size.
 (a) congruent (b) similar (c) equal (d) None of these
9. Example of similar _____ :
 (a) scalene triangle (b) acute angle triangle
 (c) equilateral triangle (d) obtuse angle triangle
10. Symbol used for Similarity:
 (a) \equiv (b) \sim (c) $=$ (d) \neq
11. $\triangle ABC$ and $\triangle DEF$ are similar than written as symbolically.
 (a) $\triangle ABC \equiv \triangle DEF$ (b) $\triangle ABC \sim \triangle DEF$ (c) $\triangle ABC > \triangle DEF$ (d) $\triangle ABC < \triangle DEF$

Unit-9

SIMILAR FIGURES

Short Questions

Exercise 9.1

1. Define polygon.
Ans: Polygon: Three or more than three sided closed figure is called polygon.
2. Define similar polygons.
Ans: Similar polygons: Two polygons are said to be similar if their corresponding angles are equal and the lengths of their corresponding sides proportional.
3. Define similar triangle.
Ans: Similar triangle: Two triangles are similar if their corresponding angles are equal and their corresponding sides are proportional. Similar triangles are same in shape but different in size.
4. Define regular polygon.
Ans: Regular polygon: A regular polygon is a polygon in which all sides are equal in length and all interior angles are equal in measure. Examples include equilateral triangles, squares, and regular hexagons.
5. Define congruent figures.
Ans: Congruent figures: Two figures are congruent if they have the same shape and size. This means their corresponding angles are equal and their corresponding sides are equal in length.
6. Define parallelogram.
Ans: Parallelogram: A parallelogram is a four-sided polygon (quadrilateral) in which opposite sides are equal in length and parallel to each other. The opposite angles of a parallelogram are also equal.
7. Define isosceles triangle.
Ans: Isosceles triangle: An isosceles triangle is a triangle that has two equal sides and two equal angles opposite to these sides.
8. Define equilateral triangle.
Ans: Equilateral triangle: An equilateral triangle is a triangle in which all three sides are of equal length, and all three interior angles measure 60° .
9. Given that $\triangle ABC$ and $\triangle DEF$ are similar, with a scale factor of $k = 3$. If the area of $\triangle ABC$ is 50 cm^2 , find the area of triangle $\triangle DEF$?
Solution:
 $A_1 = 50 \text{ cm}^2$, $k = 3$, $A_2 = ?$
 Use formula

$$\frac{A_1}{A_2} = k^2$$

$$\frac{50}{A_2} = (3)^2 \quad \Rightarrow \quad \frac{50}{A_2} = 9$$

$$\frac{50}{9} = A_1 \Rightarrow 5\frac{5}{9} = A_1$$

So, area of $\triangle DEF = 5\frac{5}{9} \text{ cm}^2$

5. Three similar jugs have heights 8cm, 12cm and 16cm. If the smallest jug holds $\frac{1}{2}$ litre, find the capacities of the other two.

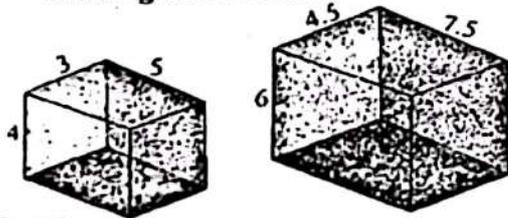
Solution:

Ratio of heights of three similar jugs = 8 : 12 : 16
The capacities are proportional to cube of their heights

$$\begin{aligned} (8 : 12 : 16)^3 &= 512 : 1728 : 4096 \\ &= 512 : 1728 : 4096 \text{ (Divide by 512)} \\ &= 1 : 3.75 : 8 \\ &= \frac{1}{2} : \frac{3.75}{2} : \frac{8}{2} \text{ (Divide by 2)} \\ &= \frac{1}{2} : 1\frac{7}{8} : 4 \end{aligned}$$

So, the capacities of other two jugs are $1\frac{7}{8}$ litres and 4 litres.

11. Find whether the solids are similar. All lengths are in cm.

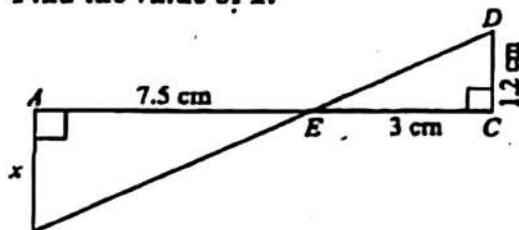


Solution:

$$\begin{aligned} \frac{4}{6} &= \frac{3}{4.5} = \frac{5}{7.5} \\ \frac{2}{3} &= \frac{30}{45} = \frac{50}{75} \\ \frac{2}{3} &= \frac{2}{3} = \frac{2}{3} \end{aligned}$$

Since the ratio of corresponding sides of the solids is equal, so the solids are similar.

12. Find the value of x:



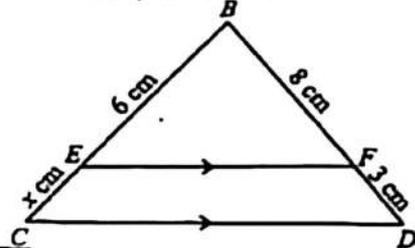
Solution:

$$\triangle ABE \sim \triangle DCE$$

So, the ratio of corresponding sides will be equal.

$$\begin{aligned} \frac{m\overline{AB}}{m\overline{DC}} &= \frac{m\overline{AE}}{m\overline{EC}} \\ \frac{x}{1.2} &= \frac{7.5}{3} \\ x &= \frac{75}{30} \times \frac{12}{10} \\ x &= \frac{12}{4} = 3 \text{ cm} \end{aligned}$$

13. Find the value of x:



Solution:

In triangle BCD, $\overline{EF} \parallel \overline{CD}$. So, \overline{EF} divides \overline{BC} and \overline{BD} into same ratio

$$\overline{BE} : \overline{EC} = \overline{BF} : \overline{FD}$$

$$\frac{\overline{BE}}{\overline{EC}} = \frac{\overline{BF}}{\overline{FD}}$$

$$\frac{6}{x} = \frac{8}{3}$$

$$\frac{3}{8} \times 6 = x \Rightarrow \frac{9}{4} = x$$

$$x = 2.25 \text{ cm}$$

14. Find the unknown value in the following:



Solution:

Since two pairs of corresponding angles are equal i.e., triangles are similar. We use the formula for ratio of areas of similar figures.

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 2.4 \text{ cm}$, $\ell_2 = 1.5 \text{ cm}$, $A_2 = 25 \text{ cm}^2$, $A_1 = ?$

$$\frac{A_1}{25} = \left(\frac{2.4}{1.5}\right)^2$$

$$\frac{A_1}{25} = \left(\frac{8}{5}\right)^2$$

$$A_1 = \frac{64}{25} \times 25 = 64 \text{ cm}^2$$

Exercise 9.2

15. Find the ratio of the areas of similar figure if the ratio of their corresponding lengths are: 1 : 3

Solution:

$$1 : 3$$

Ratio of the corresponding lengths = 1 : 3

Ratio of the areas of similar figures = ?

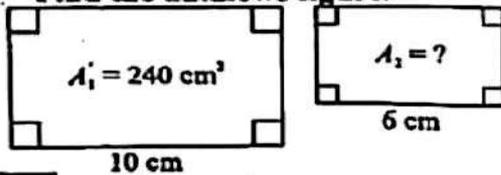
Use the formula for ratio of areas of similar figures

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$A_1 : A_2 = 1 : 9$$

16. Find the unknowns figure.



Solution:

Use the formula for ratio of areas of similar figures because two pairs of corresponding angles are equal, triangles are similar

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 10\text{cm}$, $\ell_2 = 6\text{cm}$,

$$A_1 = 240\text{cm}^2, \quad A_2 = ?$$

$$\frac{240}{A_2} = \left(\frac{10}{6}\right)^2$$

$$\frac{240}{A_2} = \frac{100}{36} \Rightarrow \frac{36}{100} \times 240 = A_2$$

$$\frac{432}{5} = A_2$$

$$86.4 = A_2$$

$$A_2 = 86.4\text{ cm}^2$$

17. The areas of two similar triangles are 16cm^2 and 25cm^2 . What is the ratio of a pair of corresponding sides?

19. The radii of two spheres are in the ratio 3 : 4. What is the ratio of their volumes?

Solution:

The ratio of the radii of two spheres = 3 : 4

Let the volume of small sphere = V_1

Let the volume of big sphere = V_2

Using formula for areas of the similar figures:

Solution:

Area of a triangle (A_1) = 16cm^2

Area of other triangle (A_2) = 25cm^2

Use the formula for ratio of areas of similar figure.

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

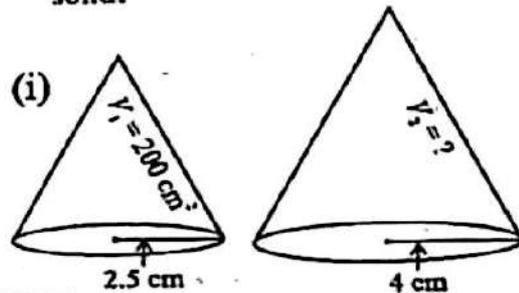
$$\frac{16}{25} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\Rightarrow \sqrt{\frac{16}{25}} = \sqrt{\left(\frac{\ell_1}{\ell_2}\right)^2}$$

$$\frac{4}{5} = \frac{\ell_1}{\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{4}{5}$$

$$\ell_1 : \ell_2 = 4 : 5$$

18. Find the unknown volume in the similar solid:



Solution:

Using formula $\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$

$$\frac{200}{V_2} = \left(\frac{2.5}{4}\right)^3$$

$$\frac{200}{V_2} = \left(\frac{5}{8}\right)^3$$

$$V_2 = 200 \times \frac{512}{125}$$

$$V_2 = 819.2 \times \text{cm}^3$$

Exercise 9.3

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$\Rightarrow V_1 : V_2 = 27 : 64$$

20. Two regular tetrahedrons have volume in the ratio 8 : 27. What is the ratio of their sides?

Solution:

The ratio of the volumes of two regular tetrahedron = 8 : 27

Using formula for areas of the similar figures:

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{8}{27} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{2^3}{3^3} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\left(\frac{2}{3}\right)^3 = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\sqrt[3]{\left(\frac{2}{3}\right)^3} = \sqrt[3]{\left(\frac{\ell_1}{\ell_2}\right)^3}$$

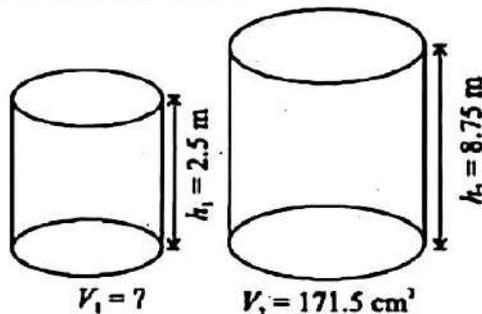
$$\frac{2}{3} = \frac{\ell_1}{\ell_2}$$

$$\frac{\ell_1}{\ell_2} = \frac{2}{3} \Rightarrow \ell_1 : \ell_2 = 2 : 3$$

Unit-9

SIMILAR FIGURES**Long Questions**

- The radii of two spheres are in the ratio 3 : 4. What is the ratio of their volumes?
- Two regular tetrahedrons have volume in the ration 8 : 27. What is the ratio of their sides?
- The areas of two similar triangles are 16cm² and 25cm². What is the ratio of a pair of corresponding sides?
- The areas of two similar triangles are 144cm² and 81cm². If the base of the large triangle is 30cm, find the corresponding base of the smaller triangle.
- Two right cones have volumes in the ration 64 : 125. What is the ratio of:
 - their heights
 - their base areas?
- Find the missing value in similar solids.



- Two similar bottles are such that one is twice as high as the other. What is the ratio of their surface areas and their capacities?
- Three similar drinking glasses have heights 7.5 cm, 9 cm and 10.5 cm. If the tallest glass holds 343 millilitres, find the capacities of the other two.
- Find the values of x, y and z in the given figure.

